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ON AUTOMATIC CONTROL

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on Automatic Control

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Standard Terminology for Feedback Control

For a number of years it has been recognized that there is a need for standard automatic control terminology. In the past, AIEE, ASME, IRE, and ASA Committees have been working toward this goal in an effort to make technical reading and writing easier and to eliminate confusion in the meaning of terms which are used to describe system performance.

With the increasing use of feedback control and with the demand for higher performance, it is becoming more important that written specifications be interpreted properly to ascertain exactly what system performance is desired by a customer. There have been frequent misunderstandings about common terms such as bandwidth, M peak, phase margin, damping, rise time, overshoot, settling time, etc. Actually, a system may be constructed to specifications and yet not meet the customer's performance requirements. Of course, it would be desirable to state that the terms referred to in a specification are defined in an accepted, published standard.

It is not a function of the PGAC to produce such standards for the IRE. This is a responsibility of the IRE Committee 26.0 on feedback control and its subcommittees 26.1 which considers standard terminology for defining a system, and 26.2 which considers standards for measuring system characteristics. Committee 26.2 is relatively new, and it has not published any standards for measurement. However, 26.1 has produced a list of standard symbols and terminology which was approved by committee 26.0 and eventually by the IRE to become the first feedback control standards to receive official sanction by a major engineering organization. These standards were published in the November, 1955 and January, 1956 issues of PROCEEDINGS and in the PGAC TRANSACTIONS of February, 1955. They evolved from a concentrated study of other existing and proposed standards Some were adopted with little change, others were modified considerably, and new definitions were proposed if necessary. It was the goal of the committee to create standards which could be used precisely for a large variety of systems, particularly in electronic control where the dc loop gain may be zero. However, the process of creating standards is not easy. It is a difficult, tedious task. Varied opinions are not easily blended into a satisfactory sentence. However, the experience is educational, especially when an attempt is made to modify a favorite expression of a committee member.

Although it is difficult for a few committee members to create a definition acceptable to all, it is much more difficult to establish a standard which will be accepted throughout the control field as the definition of a phrase in a textbook or in a specification. For this reason, the committee intends to publish future proposed standards on automatic control in these TRANSACTIONS solely for the purpose of provoking comment and constructive criticism. In this way, it is hoped that the standard may be better defined before it is finally approved by the IRE feedback control committee and adopted as an official IRE standard. The first set of proposed standards on terminology is included in this issue. Suggestions for rewriting the standards, for eliminating them, or for establishing new definitions are requested from any interested person. They should be sent to the editor of these TRANSACTIONS where they will be considered for publication, if desired, and then forwarded to the subcommittee chairman.

Active assistance in this endeavor is needed, and anyone who is interested in establishing new standards definitions will be welcome to join subcommittee 26.1 or 26.2 by contacting the editor of these TRANSACTIONS.

—*The Editor*

The Issue in Brief

There are only four papers in this issue of the TRANSACTIONS, which is half of the number usually included. It would be interesting to know whether readers favor fewer papers in more TRANSACTIONS or fewer TRANSACTIONS with more papers. In addition to the papers, the annual directory of automatic control engineers is included along with Administrative Committee activities and a proposed standard terminology for feedback control systems.

A brief review of each paper follows.

How the Bandwidth of a Servo Affects Its Saturated Response, **George A. Biernson**—Page 3

All realizable servomechanisms saturate when large step inputs are applied. The response of the servo during saturation differs from the predicted linear response, and sometimes this may be detrimental to system performance. It is desirable to know when the saturation effects may be tolerated. This paper shows not only how the saturated response can be predicted from system parameters, but how system parameters can be selected to make the saturated response allowable for a step input which causes velocity and acceleration saturation.

By reducing a complex system to a simplified form, the paper illustrates that the number of overshoots which result from a velocity saturating step response is related to the maximum acceleration, the maximum velocity, and the gain-crossover frequency (bandwidth) of the primary servo loop. To limit the number of overshoots or to produce an optimum response, the bandwidth of the servo can be adjusted. However, if this reduces the bandwidth to an excessively low value, it is possible to use nonlinear compensation to allow a stable transient for a saturating input and yet maintain a high gain and bandwidth for small, nonsaturating inputs.

Analog Study of Dead-Beat Posicast Control, **G. H. Tallman and O. J. M. Smith**—Page 14

This paper supplements an earlier paper, "Posicast Control of Damped Oscillatory Systems," by Smith in the September, 1957 issue of PROCEEDINGS. Although either paper may be understood without reference to the other, the interesting principles of posicast compensation can be more easily grasped if both papers are read. The paper in this issue describes analog tests on a system where the posicast compensation is varied from its nominal theoretical configuration to illustrate the effects of imperfect compensation that might be encountered in a practical application.

Posicast control is also described as a method of splitting the input excitation into fragments which are delayed in time before being applied to the system. When properly used, it eliminates oscillations and overshoot in the response of a servomechanism having very lightly damped poles. This can be done in a time considerably less than one cycle of the uncompensated oscillation. The frequency response of the compensated system can be made flat through and beyond the resonant frequency of the system. Compensation is considered for simple and complex systems, and for special inputs and disturbances anywhere in the forward path of the control system.

On Closed-Form Expressions for Mean Squares in Discrete-Continuous Systems, **Jack Sklansky**—Page 21

If a system is to be optimized with respect to the mean square of some variable, it is usually desirable to obtain a closed-form expression for that mean square. This has been an unsolved problem in the optimization of discrete-continuous systems where both sampled-data and continuous subsystems are used.

A method of finding the desired closed form expressions is presented in this paper. It includes a complete introduction to the subject, and the essential steps in the mathematical procedure for analyzing discrete-continuous systems with random inputs are included. The technique is illustrated by deriving closed form expressions for the output and ripple of discrete-continuous systems, and for the control error of a sampled-data feedback loop. An application to a "track-while-scan" system is also included.

Bibliography of Sampled-Data Control Systems and Z-Transform Applications, **H. Freeman and O. Lowenschuss**—Page 28

This bibliography represents an attempt to simplify the task of searching the literature to obtain a general acquaintance with the field of sampled-data control and to obtain information on certain specific aspects of it. Due to the close association of the Z-transform and sampled-data systems, a number of papers dealing exclusively with the Z transform are included in the bibliography.

The bibliography includes only that material which in the authors' opinion represents either a significant contribution to the field or has tutorial value. The bibliography is arranged alphabetically according to the name of the author, or the first-named author in cases of coauthorship. A subject index with cross references to the author list is also provided.

How the Bandwidth of a Servo Affects Its Saturated Response*

GEORGE A. BIERNSON†

Summary—**The transient response of a servomechanism to a very large step may be quite oscillatory because of saturation of the power member in acceleration. By making certain idealized yet reasonable assumptions, this paper shows that the number of overshoots of the response to a step large enough to drive the system to velocity saturation is related quite simply to the maximum acceleration, maximum velocity, and the gain crossover frequency (bandwidth) of the primary servoloop. To limit the number of overshoots in certain applications, the maximum velocity and/or bandwidth of the servo may have to be excessively lowered. However, it is possible to remedy this difficulty with nonlinear compensation, which varies the gain crossover frequency inversely with the output velocity, so as to allow a stable transient for a large step input yet maintain high gain and a wide bandwidth for small inputs.**

Introduction

An important nonlinear problem encountered in many feedback control systems is a poorly damped oscillating transient following a very large step input, which is due to saturation of the second derivative, *i.e.*, saturation of the acceleration of the controlled variable. This saturated step response occurs in many servomechanisms when the system is synchronizing on a particular reference input signal.

Although the saturated step response may be nonlinear, the important linear parameters of the system are nevertheless of fundamental importance in determining its form. The purpose of this paper is to relate in a simple fashion the form of the saturated step response to the basic linear parameters of the system and to the saturation levels of the controlled variable.

The most important linear parameter of the system is the gain crossover frequency, defined as the frequency at which the magnitude of the open loop transfer function, or loop gain, is unity. For the linear step response, the time to rise to within 37 per cent of the final value is roughly equal to the reciprocal of the gain crossover frequency. Since the loop gain of most feedback control loops approaches an asymptote inversely proportional to frequency in the region near gain crossover, it often is desirable to consider, instead of the gain crossover frequency, the more convenient parameter, the asymptote crossover frequency. That is, the frequency at which that asymptote is unity. Generally, the asymptote crossover frequency is closely equal to the actual gain crossover frequency, and consequently the asymptote crossover frequency is employed here, but is referred to for convenience as the gain crossover frequency and designated as ω_c.

To make the concepts presented here more concrete, the analysis is developed in terms of a positional servomechanism, but nevertheless can apply to any feedback system. The systems studied have saturation in both velocity and acceleration, which in a general sense represents saturation in the first and second derivatives of the controlled variable.

The impetus for this investigation was a linear analysis of the synchronizing transient of servomechanisms made by Travers.[1] He showed that in order for the drive members of a servo to keep from saturating in acceleration at the end of a large synchronizing transient, the maximum value of the system impulse response should be less than the ratio A_m/V_m, where A_m is the maximum motor acceleration and V_m the maximum velocity. Since for well-designed systems the maximum value of the impulse response is roughly equal to the gain crossover frequency[2] ω_c, this gives roughly the following requirement for linear deceleration of the synchronizing transient.

$$\omega_c < \frac{A_m}{V_m}. \tag{1}$$

Of course, it generally is desirable that some acceleration saturation occur at the end of the synchronizing transient in order to use the drive motor effectively. However, Travers reasoned intuitively that if the gain crossover frequency were increased more than a factor of ten greater than the ratio A_m/V_m, a very poor synchronizing transient would probably result. The analysis of this paper shows that Travers' reasoning was essentially correct.

The material of this paper was presented originally in a report by the author.[3]

Analysis of a Rate Feedback System

Description of System

The first system to be considered is the rate feedback system shown in Fig. 1. It is a good approximation to a large class of feedback control systems. It could represent a positional servomechanism with tachometer feedback. The variables and parameters of the system are listed.

* Manuscript received by the PGAC, May 5, 1957.
† Sylvania Electric Products, Inc., Waltham, Mass.

[1] P. Travers, "Motor Saturation in Servomechanisms," Mass. Inst. Tech., Cambridge, Mass., Dept. of Elec. Eng., Master's Thesis; September, 1948.
[2] G. A. Biernson, "Estimating transient responses from open-loop frequency response," *AIEE Trans., Applications and Industry*, pp. 388–403; January, 1956.
[3] —, "Relating the Saturated Step Response of a Servo to Its Bandwidth," M.I.T., Cambridge, Mass., Servomechanisms Lab., Rep. 7138-TM-2; June 3, 1955.

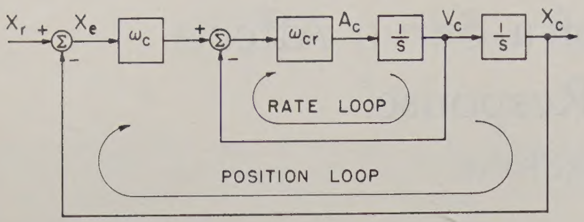


Fig. 1—Rate feedback system.

X_r = reference input (position).
X_c = controlled variable (position).
X_e = error in position.
V_c = velocity of controlled variable.
A_c = acceleration of controlled variable.
ω_c = gain crossover frequency of position loop.
ω_{cr} = gain crossover frequency of rate loop.

The absolute values of speed and acceleration are designated by the symbols V and A:

$$V = |V_c|, \tag{2}$$

$$A = |A_c|. \tag{3}$$

The maximum speed and acceleration capabilities of the controlled variable are designated as V_m and A_m:

$$V_m = \text{maximum } (V) = \text{maximum } |V_c|,$$

$$A_m = \text{maximum } (A) = \text{maximum } |A_c|.$$

The significance of the frequencies ω_c and ω_{cr} is illustrated by the plots in Fig. 2. The open loop transfer function of the rate loop (with the position loop opened) is shown by Fig. 1 to be

$$G_{(r)} = \omega_{cr}/s. \tag{4}$$

The closed loop response of the rate loop is

$$\frac{G_{(r)}}{1 + G_{(r)}} = \frac{\omega_{cr}}{s + \omega_{cr}}. \tag{5}$$

The open loop transfer function of the position loop (with the rate loop included) is

$$G_{(p)} = \frac{\omega_c}{s}\left[\frac{G_{(r)}}{1 + G_{(r)}}\right] = \frac{\omega_c}{s[(s/\omega_{cr}) + 1]}. \tag{6}$$

Frequency response plots of $|G_{(p)}|$ and $|G_{(r)}|$ are shown in Fig. 2. These are plots of the loop gains of the position and rate loops.

Ideal Saturated Step Response

The ideal saturated step response can be considered one in which the controlled variable is saturated at all times either in acceleration or velocity and which has no overshoot. Such a transient yields the fastest possible synchronizing response for the given saturation limits of the controlled member. The gain crossover frequencies ω_c and ω_{cr} required to give this ideal transient shall be determined now.

Fig. 3 shows both the position X_c (or error X_e) and velocity V_c for the ideal nonlinear response of the system of Fig. 1 to a large step input. At time t_1 the step is applied, and between t_1 and t_2 the controlled member is under maximum acceleration A_m. At t_2 the controlled variable reaches maximum velocity V_m, and between t_2 and t_3 it moves with constant velocity. Now in order for the transient to end in the shortest possible time, it is necessary for the system to start decelerating at a point 3, such that the velocity is reduced to zero at the time the error reaches zero, at point 4. Thus, between points 3 and 4 the controlled member should be under a saturated deceleration of A_m.

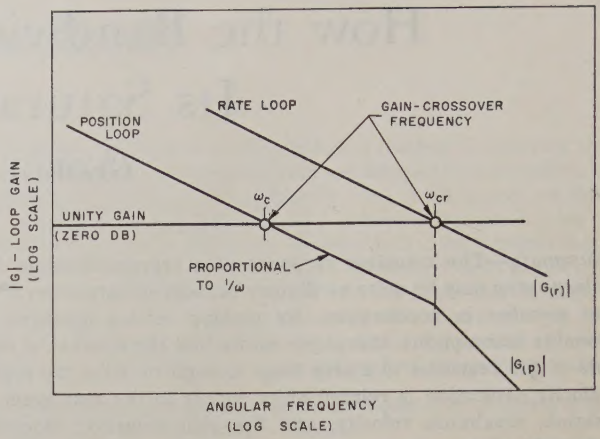


Fig. 2—Loop gains of position and rate loops.

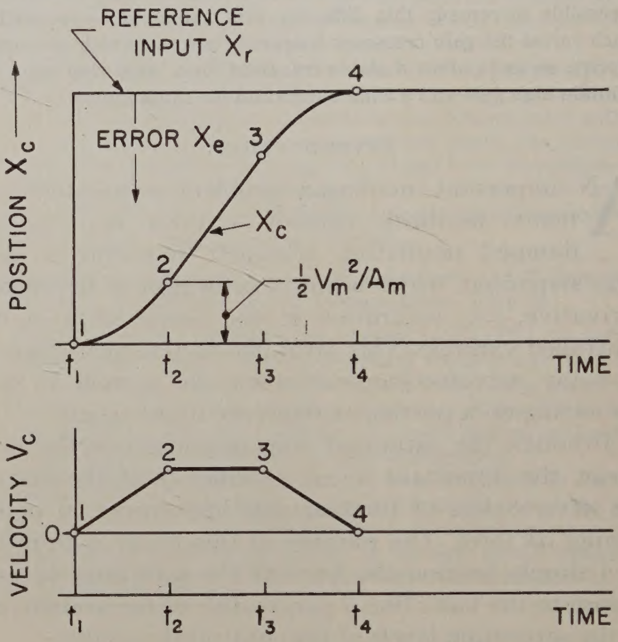


Fig. 3—Ideal saturated step response.

The time for deceleration $(t_4 - t_3)$ obviously is equal to the time for acceleration $(t_2 - t_1)$, which is equal to

$$(t_4 - t_3) = (t_2 - t_1) = \frac{V_m}{A_m}. \tag{7}$$

The motion of the controlled variable during acceleration and during deceleration is equal to $(1/2)A_m t^2$, which is equal [by (7)] to

$$\frac{1}{2} A_m t^2 = \frac{1}{2} A_m \left(\frac{V_m}{A_m}\right)^2 = \frac{1}{2} \frac{V_m^2}{A_m}. \tag{8}$$

This distance is shown in Fig. 3. Eq. (8) thus shows that, in order to achieve the ideal saturated step response for an input large enough to produce velocity saturation, the system must "switch" discontinuously to maximum deceleration at an error X_e equal to $V_m^2/2A_m$.

The manner in which the system switches to maximum deceleration can be seen by examining Fig. 1, which shows the acceleration to be equal to

$$A = \omega_{cr}(\omega_c X_e - V_c). \tag{9}$$

Eq. (9) shows that the acceleration is positive when

$$\omega_c X_e > V_c \tag{10}$$

and it is negative when

$$\omega_c X_e < V_c. \tag{11}$$

Thus, the system switches between positive and negative acceleration at an error of

$$X_e = V_c/\omega_c. \tag{12}$$

If ω_{cr} is made much greater than (A_m/V_m), (9) shows that a very small change of X_e from the value of (12) produces either maximum acceleration or deceleration (*i.e.*, $A = +A_m$ or $A = -A_m$). Thus, for large values of the rate loop crossover frequency ω_{cr}, the system switches almost instantaneously between maximum acceleration and maximum deceleration at the value of error given by (12).

Prior to the point of switching to deceleration, the controlled variable speed is equal to V_m, so that the switch to maximum deceleration occurs at the error V_m/ω_c. Therefore, in order that the saturated step response not overshoot, this switching must take place at an error larger than the distance required for the controlled member to decelerate to zero speed, which is $V_m^2/2A_m$. Hence for no overshoot

$$\frac{V_m}{\omega_c} > \frac{V_m^2}{2A_m} \tag{13}$$

which gives

$$\omega_c < \frac{2A_m}{V_m}. \tag{14}$$

For the ideal saturated step response, ω_c should be equal to $2A_m/V_m$.

Saturated Step Response with Overshoot

If ω_c is increased above the value $2(A_m/V_m)$, overshoot occurs in the saturated step response. An analysis presented in Appendix I shows that the number of overshoots n of the saturated step response is equal to

$$n = \frac{V_m \omega_c}{2A_m} - 1. \tag{15}$$

When n is not an integer, the number of overshoots is the next highest integer. Solving (15) for ω_c gives the convenient expression

$$\omega_c = \frac{2A_m}{V_m}(1 + n). \tag{16}$$

When n is set equal to zero, (16) reduces to the optimum value for ω_c, which is $2A_m/V_m$.

The settling time of the saturated step response is

$$T_s = (V_m/A_m)(1 + n). \tag{17}$$

This does not include the time during which the controlled member is moving at saturated velocity (the time between points 2 and 3 in Fig. 3). The minimum size of the step input required to produce a full saturated step response (*i.e.*, to produce saturated velocity) is

$$X_{c(\mathrm{sat})} = (n + 2)(V_m/\omega_c) = \left(\frac{n+2}{n+1}\right)\frac{V_m^2}{2A_m}. \tag{18}$$

The peak overshoot is related to this minimum input step by

$$P_{\mathrm{ov}} = [n/(n + 2)]X_{c(\mathrm{sat})}. \tag{19}$$

The above equations apply strictly only for integer values of n. A noninteger value for n indicates that in the last segment of the transient the system operates linearly with respect to the position loop, *i.e.*, with a velocity proportional to error rather than with a constant acceleration.

Effect of Rate Loop Crossover Frequency

The above analysis has assumed that the gain crossover frequency of the rate loop ω_{cr} is very much greater than the ratio (A_m/V_m):

$$\omega_{cr} \gg (A_m/V_m). \tag{20}$$

Substituting (16) into (20) gives an alternate expression for this requirement:

$$\omega_{cr} \gg \omega_c/2(1 + n). \tag{21}$$

An approximate analysis of the effect of a low value of ω_{cr} is made in Appendix II. The analysis shows that the value of ω_c for no overshoot is approximately

$$\omega_c \cong \frac{2A_m}{V_m}\left(1 - \frac{\omega_c}{4\omega_{cr}}\right). \tag{22}$$

If ω_{cr} is infinite, ω_c is equal to the value $2A_m/V_m$ calculated previously in (16) when n is set equal to zero. As ω_{cr} is decreased from infinity, ω_c must be decreased if there is to be no overshoot.

Usually the gain crossover frequency of the rate loop is at least twice that of the position loop,

$$\omega_{cr} > 2\omega_c. \tag{23}$$

For this condition, the value for ω_c in (22) is no less than 7/8 of the value it has for infinite ω_{cr}. Thus, as long as the rate loop has a gain crossover frequency at least

twice that of the position loop, one can assume that ω_{cr} is infinite without making much error in calculating the characteristics of the saturated step response.

Effect of Other Dynamic Elements in the System

Additional high-frequency lags in the systems, which always occur in practice, would not alter the saturated step response appreciably, as long as the systems are adequately stable under linear conditions. The primary effect of such lags on the saturated step response is to delay the switching and hence somewhat prolong the transient.

On the other hand, low-frequency integral networks in the system have a very pronounced effect upon the saturated step response. This effect in some cases is so bad that it is necessary to short the integral networks while synchronizing. Such shorting, of course, does not limit the accuracy of the system, because integral networks are not designed to operate under such conditions. Leaving the integral networks in the system during this transient not only tends to produce instability, but also adds a tail to the transient.

Extension to Other Systems

Besides the rate feedback system described in the preceding section. there are two other common types of systems to which the analysis can apply if suitably modified. These are the velocity source system illustrated in Fig. 4 and the lead network system in Fig. 5(a). The lead network system uses a lead network for compensation rather than rate feedback. The velocity source system needs no compensation, because the power member provides a velocity proportional to applied signal. An example of a typical velocity source system is a hydraulic servo employing a valve which gives an output flow proportional to valve displacement.

Velocity Source System

For the system of Fig. 4, the nondimensionalized variable E_m represents the signal applied to the motor. The torque-speed curve of the motor shows that for a given value of E_m the motor supplies whatever acceleration (*i.e.*, torque) is required, up to its maximum acceleration capability, to drive the controlled variable at a velocity proportional to E_m. Therefore, the system acts just like a rate feedback system (Fig. 1) with an infinite gain crossover frequency in the rate loop. The velocity source system is under maximum positive acceleration whenever $\omega_c X_e$ is greater than V_c, and is under maximum negative acceleration when $\omega_c X_e$ is less than V_c. Hence, the equations that have been derived for the rate feedback system of Fig. 1 apply directly to the velocity source system of Fig. 4.

Lead Network System

In Fig. 5(a), the lead network transfer function is shown as $(s+\omega_l)$, which approximates the actual transfer function of a lead network:

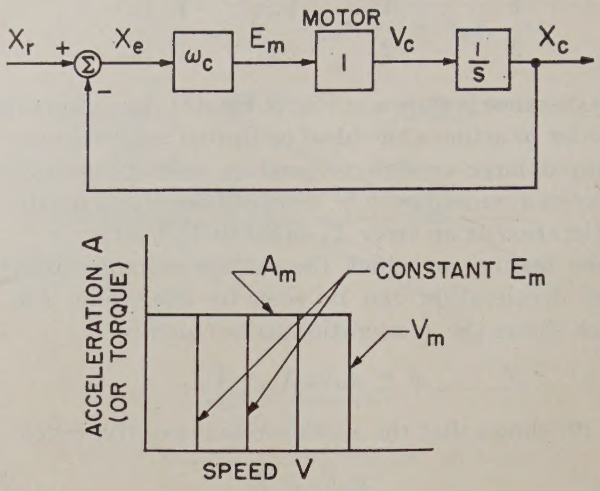
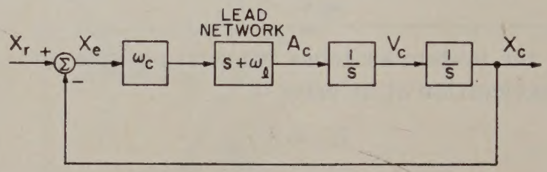
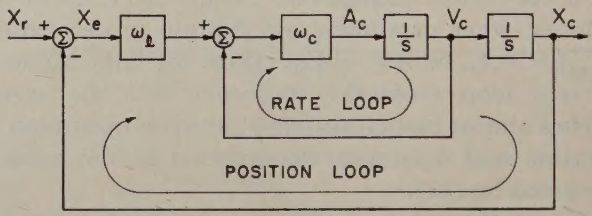


Fig. 4—Velocity source system.



(a)



(b)

Fig. 5—Lead network compensated systems. (a) Lead network system. (b) Equivalent rate feedback system.

$$\frac{s+\omega_l}{s+\alpha\omega_l} = \frac{(1/\alpha)(s+\omega_l)}{(s/\alpha\omega_l+1)} \tag{24}$$

in cascade with a gain of α. The block diagram neglects the factor $[(s/\alpha\omega_l)+1]$, but this factor has little effect upon the saturated step response if α is reasonably large.

An analysis of the lead network system is presented in Appendix III. The analysis shows that during the saturated step response the lead network system of Fig. 5(a) behaves essentially the same as the equivalent rate feedback system in Fig. 5(b). The equivalent rate feedback system has a position loop crossover frequency equal to ω_c and a rate loop crossover frequency equal to ω_l. Applying (16) and (23) derived for the rate feedback system gives

$$\omega_l = \frac{2A_m}{V_m}(1+n) \tag{25}$$

provided that

$$\omega_c > 2\omega_l. \tag{26}$$

The condition of (26) is almost always satisfied in practice.

Thus, the equations derived for the rate feedback system apply to the lead network system when ω_c for the rate feedback system is replaced by the lead network break frequency ω_l.

Nonlinear Techniques for Improving Performance

It has been shown that limitations must be placed upon the position loop gain in order to achieve adequate synchronizing performance. In many cases, however, these restrictions result in a great decrease in the linear performance of the system, and hence it is very desirable to employ nonlinear techniques to allow both a fast linear response and an adequately damped saturated step response.

The principle for achieving this is based upon the equation for the ideal saturated step response obtained by setting n equal to zero in (16).

$$\omega_c = \frac{2A_m}{V_m}. \tag{27}$$

The equation shows that, for an ideal step response to a saturating step input, the gain crossover frequency should be inversely proportional to the maximum speed of the controlled variable. Hence, ideal performance can be obtained for all operating speeds if ω_c is varied inversely with the speed. Of course, this variation of ω_c could not be achieved for very small speeds because it would require an excessively large ω_c, which would lead to instability because of linear effects. Consequently, when the system is operating at low speed, ω_c should be set at the maximum value commensurate with adequate linear stability, and when the speed increases above the value $2A_m/\omega_c$ the frequency ω_c should be decreased inversely proportional to the speed.

A simple way to instrument this change of ω_c in a rate feedback system is to place a quadratic gain element in the rate feedback path,[4] so that the gain in this path is proportional to the magnitude of the velocity signal. This has the effect of changing the gain in the loop (and hence the gain crossover frequency ω_c) inversely with the controlled variable speed. On the other hand, it has the disadvantage of varying the gain cross over frequency in the tachometer loop, so that, if the tachometer loop is to be kept stable at high speed, it must operate at excessively low bandwidth at low speed.

Another way of improving the stability of the saturated step response without excessively lowering the gain crossover frequency is to limit the maximum motor speed V_m by placing a governor on the motor.[5] This, of course, has the disadvantages of greatly slowing down the synchronizing transient, but it affords a simple means of stabilizing a low-performance feedback control system.

[4] W. H. Surber, Jr., "A nonlinear compensating configuration for saturating servomechanisms (abstract)," 1955 IRE Convention Record, Part 4, p. 23.

[5] W. C. Robinette, "A packaged servomechanism," *Electronics*, January, 1948.

Effect of Motor Break Frequency on Saturated Step Response

The equation relating the gain crossover frequency to the number of overshoots on the saturated step response can be expressed in terms of the motor break frequency (or its reciprocal, the motor time constant), provided the motor break frequency (or time constant) is defined on a nonlinear basis. To show this, it is necessary to review the commonly used linear definition for the motor break frequency.

If the torque-speed curve of the motor is linear as shown in Fig. 6(a), (28) holds for a constant value of motor voltage E_m.

$$T_s = f_m V + J_m A \tag{28}$$

$$= (f_m + sJ_m)V, \tag{29}$$

where T_s is the stalled torque (torque at zero speed), J_m is the motor inertia (including load), and f_m is the negative of the slope of the torque-speed curve, that is,

$$f_m = -\frac{dT}{dV} \text{ for constant } E_m. \tag{30}$$

The stalled torque generally is nearly proportional to the applied motor voltage E_m, so that one can assume the relation

$$T_s = K_m E_m, \tag{31}$$

where the proportionality constant K_m may be called the motor gain. Combining (29) and (31) gives for the transfer function between applied voltage and motor speed

$$\frac{V}{E_m} = \frac{K_m}{sJ_m + f_m}. \tag{32}$$

By defining the motor time constant τ_m as

$$\tau_m = J_m/f_m, \tag{33}$$

this transfer function may be expressed in the nondimensional form

$$\frac{V}{E_m} = \frac{K_m/f_m}{\tau_m s + 1}. \tag{34}$$

In many cases, however, the torque-speed curves are not linear and hence the above analysis does not strictly apply. For example, Fig. 6(b) shows a typical torque-speed curve of a two-phase servomotor driven from a high-impedance source, such as a pair of 6L6 vacuum tubes. In considering the transfer function of the motor one must generally include the power amplifier and motor together as a single unit, because of interaction between the two, and hence the voltage E_m should be taken as the input signal to the power amplifier rather than the voltage applied to the motor. The torque-speed

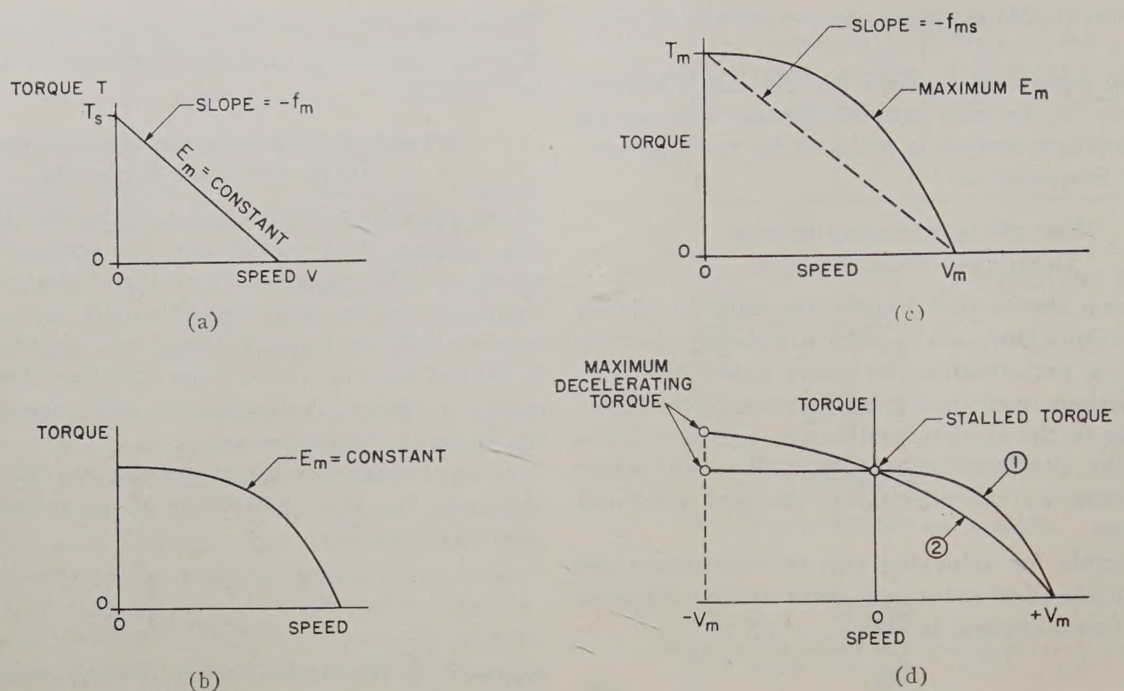


Fig. 6—Torque-speed curves. (a) Linear torque-speed curve. (b) Nonlinear torque-speed curve. (c) Definition of saturated damping factor. (d) Comparison of decelerating torque with stalled torque.

curves of a two-phase servomotor driven from a low-impedance source would be much more linear than those of Fig. 6(b).

For nonlinear torque-speed curves the same transfer function derived on a linear basis is still used, but the damping ratio f_m, defined as the slope of the torque-speed curve, is considered as a variable depending upon where the motor is operating. Since operation near zero speed is often considered to be the most important, the damping factor generally is taken as the negative of the slope of the torque-speed curve at zero speed.

On the other hand, this linearizing approach leads to the following paradox. The gain factor K_m/f_m in (34) is inversely proportional to the motor damping and hence is inversely proportional to the negative of the slope of the torque-speed curve. For the torque-speed curve in Fig. 6(b), this slope can vary from zero at low speed to a fairly large negative value at high speed, and hence the gain factor K_m/f_m of the motor can vary from a rather low value at high speed to infinity at low speed. Since this gain factor is a cascade element in the loop, this great variation in motor damping must produce a great variation in the loop gain, which may lead one to wonder how the system can have reasonable dynamic performance for all values of motor speed.

The answer to this paradox can be seen quite simply by rewriting the motor transfer function of (34) in the form

$$\frac{V}{E_m} = \frac{K_m/J_m}{s + \omega_m}, \tag{35}$$

where ω_m is the motor break frequency, which is the reciprocal of the motor time constant τ_m:

$$\omega_m = \frac{1}{\tau_m} = \frac{f_m}{J_m}. \tag{36}$$

The most important transient test of the dynamic performance of a feedback control loop is its step response, and the rise time and peak overshoot (or stability) of the step response are determined primarily by the characteristics of the loop transfer function in the region near gain crossover. Hence variations in the motor transfer function V/E_m are of basic importance only if they occur in the frequency region near gain crossover.

Systems with torque-speed curves like Fig. 6(b), generally have compensation to provide system damping, so that the resultant gain crossover frequency ω_c is much larger than the motor break frequency ω_m. For this condition the motor transfer function of (35) can be closely approximated as

$$\frac{V}{E_m} = \frac{K_m/J_m}{j\omega + \omega_m} \cong \frac{K_m/J_m}{j\omega} = \frac{K_m/J_m}{s} \tag{37}$$

in the frequency region near gain crossover. Thus, variations in the motor damping are of little importance, because the motor break frequency has negligible effect on the loop transfer function near gain crossover.

Of course, if the system does not have additional compensation to give damping, the motor damping is very important. For such cases the approximation of (37) cannot apply at gain crossover, because instability results if the gain crossover frequency is made much larger than the motor break frequency ω_m, and therefore it often is more convenient to express the motor transfer function in terms of the motor time constant τ_m as in (34). On the other hand, when there is no damping supplied by compensation, the motor damping must

be quite large for all values of speed and rather constant in order to achieve reasonable dynamic performance. For this condition, large nonlinearity of the motor damping generally is not encountered in practice.

For wide bandwidth systems in which the damping is provided by compensation, a variation in motor damping merely changes the velocity constant, that is, it changes the steady-state errors due to constant velocity inputs. For the purpose of minimizing these steady-state errors it is desirable that the motor damping be as low as possible. If the motor damping is zero at low speeds, the velocity constant is infinite, and the steady-state errors for low velocity inputs are zero.

Thus, it has been shown that the term "motor-break-frequency ω_m," or its reciprocal, the "motor time-constant τ_m," defined on a linear basis, has little effect upon system performance unless the system is operated at a rather low bandwidth, with the motor damping furnishing the main damping for the system. On the other hand, there is a similar parameter defined on a nonlinear basis which is quite important, and this parameter shall be called the *saturated motor break-frequency* ω_{ms}.

The definition for the saturated motor break-frequency ω_{ms} is illustrated in Fig. 6(c); it shows the torque-speed curve for the maximum voltage E_m. The maximum motor speed is shown as V_m and the maximum motor torque as T_m. If a straight line is drawn from the points of maximum torque and maximum speed, as shown, the negative of the slope of this line is defined as the *saturated motor damping factor* f_{ms}, so that

$$f_{ms} = T_m/V_m. \tag{38}$$

The saturated motor break-frequency ω_{ms} then may be defined as

$$\omega_{ms} = \frac{f_{ms}}{J_m} = \frac{T_m/J_m}{V_m}. \tag{39}$$

One can also define a saturated motor time-constant τ_{ms} as being equal to the reciprocal of ω_{ms}. Newton[6] has defined the "motor time-constant" in this manner, and has shown its importance in describing the adequacy of the power capabilities of a motor in various applications. He points out that the saturated motor time-constant τ_{ms} represents the time for the motor to accelerate from zero to maximum speed.

By means of the basic relation of (16) between ω_c and the number of overshoots of the saturated step response, the gain crossover frequency ω_c can be related to the saturated motor break frequency. For positive speeds, the maximum motor acceleration occurs at zero speed and is equal to

$$A_m = T_m/J_m. \tag{40}$$

However, in terms of the saturated step response it is the deceleration capabilities of the motor that are of prime importance, that is, the acceleration at negative speeds.

[6] G. C. Newton, Jr., "What size motor?" *Machine Design*, November, 1950.

If the torque-speed curve is as shown by curve 1 of Fig. 6(d), the maximum deceleration is the same as the value given in (40), but if it is as shown by curve 2 the maximum deceleration is somewhat larger. However, for simplicity, one can take (40) as representing a reasonable approximation to A_m. Combining (39) and (40) relates the maximum acceleration to the saturated motor break-frequency by

$$\omega_{ms} = A_m/V_m. \tag{41}$$

Now, the basic equations for the saturated step response have been given in (16) and (25) as

$$\omega_c = \frac{2A_m}{V_m}(1+n) \quad \begin{cases}\text{rate feetback system}\\ \text{velocity source system,}\end{cases} \tag{42}$$

$$\omega_l = \frac{2A_m}{V_m}(1+n) \quad \text{lead network system.} \tag{43}$$

Substituting (41) into the above gives

$$\omega_c = 2(1+n)\omega_{ms} \quad \begin{cases}\text{rate feedback system}\\ \text{velocity source system,}\end{cases} \tag{44}$$

$$\omega_l = 2(1+n)\omega_{ms} \quad \text{lead network system.} \tag{45}$$

These equations thus show that the saturated motor break frequency is an important limitation on system bandwidth.

It is interesting to point out that if the torque-speed curves are linear as shown in Fig. 6(a), the saturated motor break-frequency ω_{ms} is equal to the linear motor break-frequency ω_m. This illustrates why the motor break-frequency ω_m has been found to be such a limitation on system bandwidth when the torque-speed curves are linear, even when additional compensation is added to provide damping.

Conclusion

Because of saturation of the second derivative (acceleration) of the controlled variable of a system, the response to a very large step input can be very poorly damped, and may even be unstable. This nonlinear instability is due to the inability of the motor to decelerate the controlled variable adequately as the error reaches zero, so that the controlled variable is driven well into saturation in the reverse direction before coming to a stop. To remedy this problem, either the maximum speed of the controlled variable must be limited or the loop gain must be decreased, in order that the controlled variable start decelerating at a larger value of error.

In systems where instability of the saturated step response is a problem, nonlinear compensation can be employed to allow both high gain during linear operation and a fast well-damped response to large saturating step inputs. This is done by varying the loop gain inversely with the speed of the controlled variable.

For a system employing a velocity-source power member (such as is used in many hydraulic servomech-

anisms) the gain crossover frequency ω_c is related as follows to the number of overshoots n of the step response for an input large enough to produce maximum saturation:

$$\omega_c = \frac{2A_m}{V_m}(1+n), \tag{46}$$

where A_m is the maximum motor acceleration and V_m the maximum velocity. This same relation also applies to rate feedback compensated systems, employing an acceleration (torque) source power member, if the rate loop gain crossover frequency ω_{cr} is much larger than the position loop gain crossover frequency ω_c. Excluding the time the controlled variable is slewing at saturated velocity, the setting time for the transient is

$$T_s = \frac{2}{\omega_c}(n+1)(n+2) = (n+2)\frac{V_m}{A_m}. \tag{47}$$

For lead network compensated systems, in which the gain crossover frequency is significantly greater than the lead network break-frequency ω_l, the above relations apply if ω_c is replaced by ω_l.

Appendix I

Analysis of Oscillatory Step Response

When the gain crossover frequency ω_c of the rate feedback system is made much greater than $2A_m/V_m$, the saturated step response becomes quite oscillatory. This Appendix presents an analysis of this oscillatory step response.

Fig. 7 shows an oscillatory saturated step response of the rate feedback system of Fig. 1. Points 2, 4, 6, and 8 represent the points where switching of acceleration occurs, and hence represent points of maximum speed; while points 1, 3, 5, 7, and 10 represent points of zero speed. For this transient the value of ω_c has been set so that following the last point of acceleration (point 2) the system decelerates with maximum deceleration, reaching zero velocity at zero error.

In the analysis to be presented, the following symbols are employed:

X_n = value of X_c at point n.
$X_{e(n)}$ = value of X_e at point n.
V_n = speed at point n.

At points 2, 4, 6, and 8, where the acceleration switching occurs, (12) shows that the magnitude of error is related to the speed by

$$|X_{e(n)}| = V_n/\omega_c. \tag{48}$$

This gives

$$|X_{e(2)}| = V_2/\omega_c \tag{49a}$$
$$|X_{e(4)}| = V_4/\omega_c \tag{49b}$$
$$|X_{e(6)}| = V_6/\omega_c \tag{49c}$$
$$|X_{e(8)}| = V_8/\omega_c. \tag{49d}$$

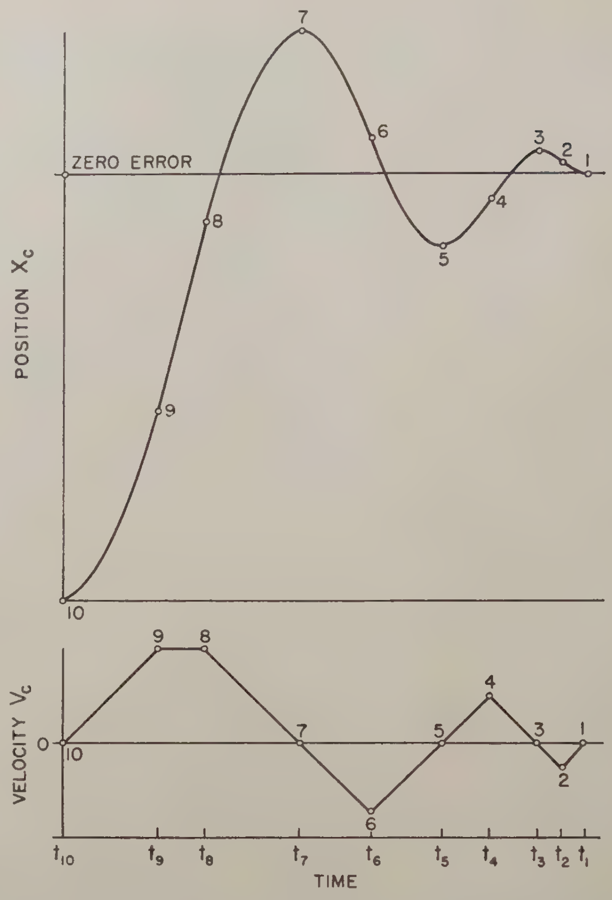


Fig. 7—Saturated step response with overshoot.

Fig. 7 shows that the magnitudes of error at successive switching points are related as follows.

$$|X_{e(2)}| = |X_2 - X_1| \tag{50a}$$
$$|X_{e(4)}| = |X_4 - X_3| - |X_3 - X_2| - |X_{e(2)}| \tag{50b}$$
$$|X_{e(6)}| = |X_6 - X_5| - |X_5 - X_4| - |X_{e(4)}| \tag{50c}$$
$$|X_{e(8)}| = |X_8 - X_7| - |X_7 - X_6| - |X_{e(6)}|. \tag{50d}$$

During the entire transient between point 8 and point 1, full acceleration of magnitude A_m is applied, alternately positively and negatively. Hence the distances ΔX between successive points of zero and maximum speed are equal to

$$\Delta X = \frac{1}{2}A_m(\Delta t)^2 = \frac{1}{2}\frac{V^2}{A_m}, \tag{51}$$

where V is the maximum speed for that section of the transient. Applying (51) to the specific regions of the transient gives

$$|X_2 - X_1| = |X_3 - X_2| = \frac{1}{2}\frac{V_2^2}{A_m} \tag{52a}$$
$$|X_4 - X_3| = |X_5 - X_4| = \frac{1}{2}\frac{V_4^2}{A_m} \tag{52b}$$
$$|X_6 - X_5| = |X_7 - X_6| = \frac{2}{2}\frac{V_6^2}{A_m} \tag{52c}$$

$$|X_8 - X_7| = |X_{10} - X_9| = \frac{1}{2}\frac{V_8^2}{A_m}. \qquad (52d)$$

Substituting (50a)–(50d) and (52a)–(52d) into (49a)–(49d) gives

$$\frac{V_2}{\omega_c} = \frac{1}{2}\frac{V_2^2}{A_m} \qquad (53a)$$

$$\frac{V_4}{\omega_c} = \frac{1}{2}\frac{V_4^2}{A_m} - \frac{1}{2}\frac{V_2^2}{A_m} - \frac{V_2}{\omega_c} \qquad (53b)$$

$$\frac{V_6}{\omega_c} = \frac{1}{2}\frac{V_6^2}{A_m} - \frac{1}{2}\frac{V_4^2}{A_m} - \frac{V_4}{\omega_c} \qquad (53c)$$

$$\frac{V_8}{\omega_c} = \frac{1}{2}\frac{V_8^2}{A_m} - \frac{1}{2}\frac{V_6^2}{A_m} - \frac{V_6}{\omega_c}. \qquad (53d)$$

Rearranging the individual equations gives

$$V_2 = \frac{2A_m}{\omega_c} \qquad (54a)$$

$$V_4^2 - \frac{2A_m}{\omega_c} V_4 - \left[\frac{2A_m}{\omega_c} V_2 + V_2^2\right] = 0 \qquad (54b)$$

$$V_6^2 - \frac{2A_m}{\omega_c} V_6 - \left[\frac{2A_m}{\omega_c} V_4 + V_4^2\right] = 0 \qquad (54c)$$

$$V_8^2 - \frac{2A_m}{\omega_c} V_8 - \left[\frac{2A_m}{\omega_c} V_6 + V_6^2\right] = 0. \qquad (54d)$$

Solve (54b) for V_4,

$$V_4 = \frac{A_m}{\omega_c} \pm \sqrt{\left(\frac{A_m}{\omega_c}\right)^2 + \left[2\left(\frac{A_m}{\omega_c}\right)V_2 + V_2^2\right]}. \qquad (55)$$

The polynomial under the square-root sign is a perfect square. Choose the plus sign, because the speed V_4 must be positive. This gives

$$V_4 = \frac{A_m}{\omega_c} + \frac{A_m}{\omega_c} + V_2 = \frac{2A_m}{\omega_c} + V_2. \qquad (56)$$

Thus, the solution of (54a)–(54d) is

$$V_2 = 2\,\frac{A_m}{\omega_c} \qquad (57a)$$

$$V_4 = \frac{2A_m}{\omega_c} + V_2 = 2\left(\frac{2A_m}{\omega_c}\right) \qquad (57b)$$

$$V_6 = \frac{2A_m}{\omega_c} + V_6 = 3\left(\frac{2A_m}{\omega_c}\right) \qquad (57c)$$

$$V_8 = \frac{2A_m}{\omega_c} + V_6 = 4\left(\frac{2A_m}{\omega_c}\right). \qquad (57d)$$

Now the speed V_8 is the maximum speed of the system V_m. Hence (57d) gives for this transient, which has three overshoots,

$$V_m = \frac{2A_m}{\omega_c}(4). \qquad (58)$$

Hence, for the case of n complete overshoots,

$$V_m = \frac{2A_m}{\omega_c}(n+1). \qquad (59)$$

Solving for ω_c gives

$$\omega_c = \frac{2A_m}{V_m}(n+1). \qquad (60)$$

Eq. (60) is thus a generalization of (14), which applied to the ideal saturated step response. Eq. (60) shows that the greater ω_c is above $2A_m/V_m$, the greater is the number of overshoots in the saturated step response. Solving (60) for n gives

$$n = \frac{V_m\omega_c}{2A_m} - 1. \qquad (61)$$

This equation, of course, applies strictly only for integer values of n. A noninteger value for n indicates that in the last segment of the transient the system operates linearly with respect to the position loop, that is, with a velocity proportional to error rather than with a constant acceleration.

The settling time of the transient can be computed as follows. Designate the time interval between two successive points a and b as t_{ab}. The time intervals of the various segments of the transient are equal to

$$t_{12} = t_{23} = V_2/A_m \qquad (62a)$$

$$t_{34} = t_{45} = V_4/A_m \qquad (62b)$$

$$t_{56} = t_{67} = V_6/A_m \qquad (62c)$$

$$t_{78} = t_{9-10} = V_8/A_m. \qquad (62d)$$

Neglecting the time the controlled variable is slewing at maximum velocity, between points 8 and 9, the total settling time is

$$T_s = \frac{2V_2}{A_m} + \frac{2V_4}{A_m} + \frac{2V_6}{A_m} + \frac{2V_8}{A_m} \qquad (63)$$

$$= \frac{2}{A_m}(V_2 + V_4 + V_6 + V_8) \qquad (64)$$

$$= \frac{4}{\omega_c}(1 + 2 + 3 + 4). \qquad (65)$$

For a transient with n overshoots, the settling time is thus equal to

$$T_s = \frac{4}{\omega_c}[1 + 2 + \cdots + n + (n+1)]. \qquad (66)$$

Now the sum of integers up to q is equal to

$$1 + 2 + 3 + \cdots q = \frac{q(q+1)}{2}. \qquad (67)$$

Hence (66) is equal to

$$T_s = \frac{2}{\omega_c}(n+1)(n+2). \qquad (68)$$

In terms of the saturation parameters, the settling time is equal to

$$T_s = \frac{(n+2)V_m}{A_m}. \tag{69}$$

It is also important to know the size of the overshoot in the transient. The magnitudes of error at the points of zero velocity are equal to

$$|X_{e(3)}| = \frac{1}{2}\frac{V_3^2}{A} + \frac{V_2}{\omega_c} \tag{70a}$$

$$|X_{e(5)}| = \frac{1}{2}\frac{V_4^2}{A} + \frac{V_4}{\omega_c} \tag{70b}$$

$$|X_{e(7)}| = \frac{1}{2}\frac{V_6^2}{A} + \frac{V_6}{\omega_c} \tag{70c}$$

$$|X_{3(10)}| = \frac{1}{2}\frac{V_8^2}{A} + \frac{V_8}{\omega_c} + |X_9 - X_8|. \tag{70d}$$

Substituting (57a)–(57d) for the speeds in the above gives

$$|X_{e(3)}| = \frac{2A_m}{\omega_c^2}(1+1)(1) \tag{71a}$$

$$|X_{e(5)}| = \frac{2A_m}{\omega_c^2}(1+2)(2) \tag{71b}$$

$$|X_{e(7)}| = \frac{2A_m}{\omega_c^2}(1+3)(3) \tag{71c}$$

$$|X_{e(10)}| = \frac{2A_m}{\omega_c^2}(1+4)(4) + \Delta X_{\text{sat}}, \tag{71d}$$

where ΔX_{sat} is the motion of the controlled variable while under saturated velocity, given by

$$\Delta X_{\text{sat}} = |X_9 - X_8|. \tag{72}$$

Eq. (71c) represents the peak overshoot P_{ov}. For the general transient of n overshoots, the peak overshoot is thus

$$P_{\text{ov}} = \frac{2A_m}{\omega_c^2} n(1+n). \tag{73}$$

More convenient expressions are

$$P_{\text{ov}} = \frac{nV_m}{\omega_c} \tag{74}$$

$$= \frac{V_m^2}{2A}\left(\frac{n}{n+1}\right). \tag{75}$$

Eq. 74 shows that the peak overshoot is n times the maximum linear range of the position loop, which is V_m/ω_c. Eq. (75) shows that the peak overshoot varies with n by the factor $n/(n+1)$. Thus, as long as the transient has at least a couple of overshoots, the peak overshoot does not increase much with increasing n.

The maximum error occurs at the start of the transient [representing $X_{e(10)}$ for the example] and is equal to

$$X_{e(\max)} = \frac{2A_m}{\omega_c^2}(n+1)(n+2) + \Delta X_{\text{sat}}. \tag{76}$$

If ΔX_{sat}, the displacement during maximum velocity, is zero, (76) gives the error for the smallest step which will bring the controlled variable up to saturated velocity, which is

$$\frac{2A_m}{\omega_0^2}(n+1)(n+2) = \frac{(n+2)}{(n+1)}\frac{V_m^2}{2A}. \tag{77}$$

This varies only by a factor of two as the number of overshoots n is increased from zero to infinity.

Appendix II

Effect of Gain Crossover Frequency of Rate Loop

The analysis of the rate feedback system presented in Appendix I assumed that the gain crossover frequency of the rate loop was infinite. This Appendix develops a simple approximation of the effect of lowering the rate loop gain crossover frequency.

Fig. 1 showed that the acceleration A_c is equal to

$$A_c = \omega_{cr}(\omega_c X_e - V_c). \tag{78}$$

Hence, the controlled member is under maximum positive acceleration when

$$X_e > \frac{V_c}{\omega_c} + \frac{A_m}{\omega_c\omega_{cr}} \tag{79}$$

and it is under maximum negative acceleration when

$$X_e < \frac{V_c}{\omega_c} - \frac{A_m}{\omega_c\omega_{cr}}. \tag{80}$$

Fig. 8 shows approximately the saturated step response which occurs with a lower ω_{cr}, if ω_c is adjusted for the maximum value with no overshoot. At the error V_m/ω_c (point 3) the acceleration begins to become negative, but it is not until point 5 that the full negative acceleration $(-A_m)$ is applied. Hence, at point 5 the error is

$$X_{e(5)} = \frac{V_5}{\omega_c} - \frac{A_m}{\omega_c\omega_{cr}}. \tag{81}$$

If it is assumed that there is negligible change in velocity between points 3 and 5, the error at point 5 is approximately equal to

$$X_{e(5)} \cong \frac{V_m}{\omega_c} - \frac{A_m}{\omega_c\omega_{cr}}. \tag{82}$$

Thus, lowering the rate loop gain crossover frequency decreases the error at which maximum deceleration is applied by the amount $A_m/\omega_c\omega_{cr}$.

If the velocity is assumed nearly constant between points 3 and 5, the acceleration increases essentially

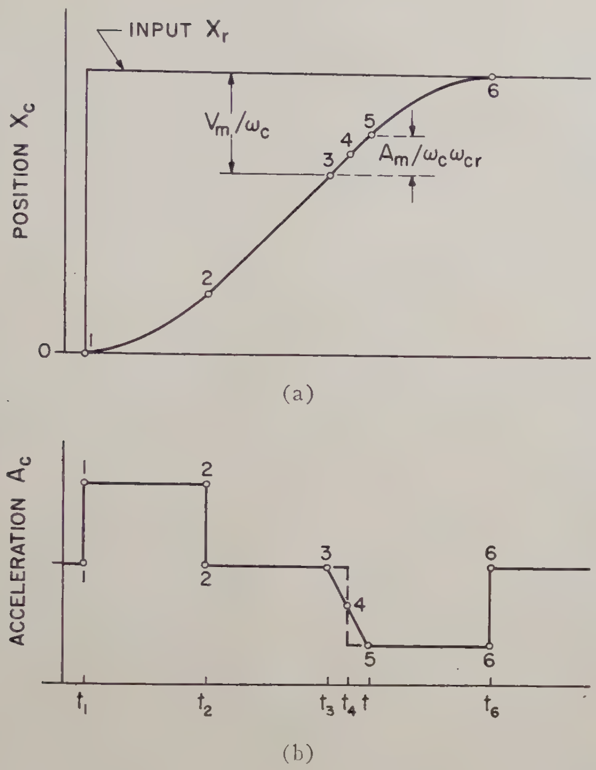


Fig. 8—Effect of rate loop gain.

linearly with the displacement in this region, as well as with time. The approximate acceleration plot is shown in Fig. 8(b). The change in velocity between points 3 and 5 is equal to the area under the acceleration curve in this region, which is equal to

$$\Delta V = \int_{t_3}^{t_5} A\, dt = \frac{1}{2} A_m(t_5 - t_3). \tag{83}$$

This same change in velocity would occur in this region if the acceleration were zero up to point 4, and were equal to A_m thereafter, where point 4 is midway between points 3 and 5. This acceleration curve is shown by the dashed segment in Fig. 8(b).

Thus, a reasonable approximation of the transient can be obtained by assuming that the acceleration switches discontinuously from zero to maximum deceleration at point 4. Since point 4 is midway between points 3 and 5, this effective switch of acceleration occurs at the error

$$X_{e(4)} = \frac{V_m}{\omega_c} - \frac{1}{2}\frac{A_m}{\omega_c\omega_{cr}}. \tag{84}$$

If the controlled variable is to decelerate from point 4 to zero velocity at zero error, the error at point 4 must be equal to

$$X_{e(4)} = \frac{1}{2}\frac{V_m^2}{A_m}. \tag{85}$$

Equating the expressions of (84) and (85) gives

$$\omega_c = \frac{2A_m}{V_m}\left[1 - \frac{2A_m/V_m}{4\omega_{cr}}\right]. \tag{86}$$

Since $(2A_m/V_m)$ is roughly equal to ω_c, (86) is roughly equal to

$$\omega_c \approx \frac{2A_m}{V_m}\left(1 - \frac{\omega_c}{4\omega_{cr}}\right). \tag{87}$$

Eq. (87) shows that, as a first approximation, the effect of decreasing ω_{cr} is to require that ω_c be decreased by the factor $[1-(\omega_c/4\omega_{cr})]$ in order for the number of overshoots of the saturated step response to remain the same.

Although the above analysis has been rather crude, it does indicate the general effect of a decrease in the rate loop gain crossover frequency. If more detailed and accurate information is desired, it is suggested that analog computer simulation be employed, because an extension of the analysis is quite cumbersome.

Appendix III

Analysis of Lead Network System

The lead network system was shown previously in Fig. 5(a). This Appendix shows that during the saturated step response the lead network system of Fig. 5(a) behaves in the same manner as the equivalent rate-feedback system of Fig. 5(b).

Fig. 5(a) shows that the acceleration A_c of the lead network system is related to the error X_e by

$$A_c = \omega_c(s + \omega_l)X_e = \omega_c(\dot{X}_e + \omega_l X_e), \tag{88}$$

where $\dot{X}_e$ is the error rate, that is, the time derivative of the error X_e. Since X_e is equal to $X_r - X_c$, the error rate is equal to

$$\dot{X}_e = \dot{X}_r - \dot{X}_c = \dot{X}_r - V_c. \tag{89}$$

Substituting (89) into (88) gives

$$A_c = \omega_c[\omega_l X_e - V_c] + \omega_c\dot{X}_r. \tag{90}$$

At the instant the step is applied, $\dot{X}_r$ is infinite; after that, $\dot{X}_r$ is zero. Thus, $\dot{X}_r$ is an impulse of area equal to the size of the step input. If this impulse is neglected, A_c is equal to

$$A_c = \omega_c[\omega_l X_e - V_c]. \tag{91}$$

This same equation applies to the tachometer system in Fig. 5(b). Thus, if the impulse term is neglected, the lead network compensated system of Fig. 5(a) behaves in the same manner as the analogous rate feedback compensated system in Fig. 5(b).

In the analogous rate feed system, the position loop gain crossover frequency is equal to the lead network break-frequency ω_l of the lead network system, and the rate loop gain crossover frequency is equal to the gain crossover frequency ω_c of the lead network system. Hence the previous equations that were derived for the rate feedback system apply to the lead network system if ω_c is replaced by ω_l. Thus, for the lead network system, the lead network break-frequency ω_l is related to the number of overshoots of saturated step response by

$$\omega_l = \frac{2A_m}{V_m}(1+n). \tag{92}$$

Now consider the $\dot{X}_r$ impulse that was neglected in (90). The lead network transfer function in Fig. 5(a) was approximated as $(s+\omega_l)$, but, as was shown by (24), is actually

$$\frac{s+\omega_l}{(s/\alpha\omega_l)+1}. \tag{93}$$

(This transfer function includes an amplifier gain factor of α.) If the effect of the lead network factor $[(s/\alpha\omega_l)+1]$ is considered, the term $(\omega_c\dot{X}_r)$ in (90) becomes

$$\frac{\omega_c\dot{X}_r}{1+(s/\alpha\omega_l)} = \omega_c\alpha\omega_l\left(\frac{sX_r}{s+\alpha\omega_l}\right). \tag{94}$$

Designate the inverse transform of this term as ΔA_c. For a step input of magnitude X, the transform of $\dot{X}_r$ is equal to X/s, and the inverse transform of (55) is

$$\Delta A_c = X\omega_c\alpha\omega_l e^{-\alpha\omega_l t}. \tag{95}$$

Assume that the step input is large enough for the system to accelerate to maximum velocity. Designate as t_a the time for the system to accelerate to maximum velocity [*i.e.*, (t_2-t_1) in Fig. 3]. The time t_a is equal to

$$t_a = V_m/A_m. \tag{96}$$

During the time t_a that the system is accelerating, the exponential term of (95) is of no concern because the motor is completely saturated anyway. This exponential merely tends to drive the motor more into saturation. At the end of this period, the term is equal to

$$\Delta A_c(t_a) = (X\omega_c\alpha\omega_l)e^{-\alpha\omega_l t_a} = (X\omega_c\alpha\omega_l)e^{-\alpha\omega_l V_m/A_m}. \tag{97}$$

Substituting (92) into (97) gives

$$\Delta A_c(t_a) = (X\omega_a\alpha\omega_l)e^{-2\alpha(1+n)}. \tag{98}$$

A reasonable value for α is 10. For no overshoots (*i.e.*, for $n=0$), the exponential factor of (98) is e^{-20}, which is equal to 2×10^{-9}. This factor is even smaller when n is greater than zero. Thus, the exponential term is essentially zero at the time t_a (when the motor has reached maximum speed).

The above discussion has justified the approximation of neglecting the $\dot{X}_r$ term in (90) when calculating the saturated step response of the lead network system. The transient due to the $\dot{X}_r$ term is of no consequence provided that the input step is sufficiently large to accelerate the motor to maximum speed. By the time the motor has come out of saturation, this transient has vanished.

Analog Study of Dead-Beat Posicast Control*

G. H. TALLMAN† AND O. J. M. SMITH†

Summary—**A method is presented which eliminates the oscillations and overshoot in a lightly damped servomechanism in a time of considerably less than one cycle of the uncompensated oscillation. The frequency response of a resonant system compensated in this manner can be made flat up through and beyond the resonant frequency of the system. For dead-beat response in one-half period, the frequency response is down only 3 db at the resonant frequency. Systems designed to have dead-beat responses in much less than one-half period have correspondingly wider bandwidths.**

The posicast method involves splitting up the input excitation into several fragments. Each fragment is applied at such a time that the sum of all the transient terms is equal to zero after the last excitation. The method is general and can be applied to electrical, mechanical, and even pneumatic systems.

Introduction

If a linear servomechanism with very lightly damped poles is excited with a step input, the usual result is many cycles of oscillation with an eventual approach to steady state. If the system is excited with only half the original excitation, the output will display the same cycles of oscillation of half amplitude before reaching the steady-state value of one half. Suppose, however, that the system is allowed to oscillate for one-half cycle, at the end of which time it is excited with a second step of one half. The two transients will be 180° out of phase and the algebraic sum will be equal to zero. However, the steady-state responses to each of the inputs add directly, and the net result is that the output reaches steady state in one-half cycle of the uncompensated oscillation.

From the above, we can easily derive the transfer function of the compensator to be placed between the input and the lightly damped system:

$$G(s) = (1+P) = k + [1-k]e^{-sT/2}. \tag{1}$$

k delivers a portion (or the first fragment) of the input excitation. The second portion, of magnitude $(1-k)$,

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arrives after a pure time wait of $T/2$ seconds, where T is the period of the oscillatory system. These two portions of the excitation are spaced one-half cycle apart, and the transient oscillations excited by the first portion are precisely and completely cancelled by the transient due to the second portion if k and $1-k$ have a ratio in accordance with the damping of the transient oscillation.

A comprehensive treatment of this type of control is given.[1] P is defined by (1) as $(G-1)$. k is a constant that always lies between one half and one. A very slightly damped system will have a k slightly greater than one half, and a heavily damped system will have a k near unity.

$$k^{-1} = 1 + \exp(\alpha T/2) \tag{2}$$

where α is the damping of the poles and where T equals the uncompensated oscillatory period. In a stable system, where α is negative, $1 \leq k^{-1} \leq 2$. The step response of this compensating network is shown in Fig. 1(a) as an initial step to which is added another step delayed by $T/2$ seconds. If the pole-zero diagram of (1) is plotted, we find that the poles are all in a vertical column at negative infinity in the s plane (practically at very high frequencies) while the zeros lie equally spaced on a line parallel to the imaginary axis.

If we set (1) equal to zero to solve for the positions of the zeros, one has the following:

$$k + (1-k)e^{-sT/2} = 0. \tag{3}$$

Separating the real and imaginary parts of the exponential with $s = \alpha + j\omega$,

$$e^{-\alpha T/2} \cos \omega T/2 = \frac{-k}{1-k} \tag{4}$$

$$je^{-\alpha T/2} \sin \omega T/2 = 0. \tag{5}$$

The imaginary part equals zero when $\sin \omega T/2$ equals zero, so that

$$|\cos \omega T/2| = 1.0. \tag{6}$$

In (4), $e^{-\alpha T/2}$ is always positive and $k/(1-k)$ is positive for $0 < k < 1.0$. Therefore, $\cos \omega T/2$ must be negative.

$$\cos \omega T/2 = -1 = (-1)^n \tag{7}$$

where n is odd.

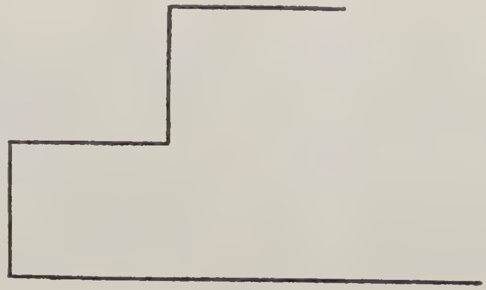
$$\omega T/2 = n\pi \tag{8}$$

where n is odd.

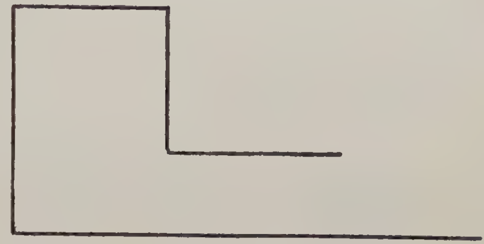
The vertical positions of the zeros in the s plane are therefore at

$$\omega = n(2\pi/T). \tag{9}$$

[1] O. J. M. Smith, "Posicast control of damped oscillatory systems," Proc. IRE, vol. 45, pp. 1249–1255; September, 1957.



(a)



(b)

Fig. 1—Step response of compensator for complex and real s-plane poles.

From (4) and (7)

$$\epsilon^{-\alpha T/2} = k/(1-k) \tag{10}$$

$$\exp(\alpha T/2) = k^{-1} + 1. \tag{11}$$

From the above, (2) can be derived. All of the zeros of G have the same α coordinate in the s plane as the α of the system poles being cancelled.

Posicast compensation can then be described as a process whereby complex zeros are generated so as to fall upon the complex poles of the system, thereby making transient oscillations impossible. A single step input to the complete system produces only a single pole at the origin. Two steps, however, appear at the input to the resonant component. After the first step, it seems as though the zeros are not present, and the system will start an oscillation. After the second step, which occurs a half cycle later, the second component of oscillation cancels the first.

An alternate, but equivalent way of looking at posicast control is to consider the component oscillations as phasors in time phase. The two oscillations are represented by two phasors displaced 180°.

After the first excitation, we have one phasor representing the first component of oscillation. After the second excitation, we have two phasors 180° apart. These phasors will cancel after the second excitation if the magnitudes are equal at $t = T/2$. To achieve this, the magnitude of the first excitation must be larger because of attenuation due to the damping.

The phasors representing the two components of oscillation are shown in Fig. 2. The first component is shown as decreasing in magnitude as it rotates counter

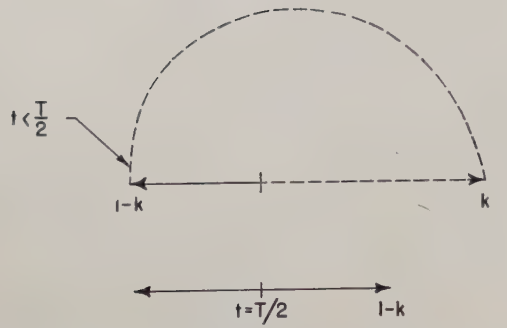


Fig. 2—Phasor diagram for the oscillatory components excited by the double-step output of the compensator. AC vectors.

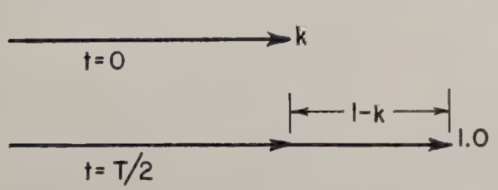


Fig. 3—Phasor diagram for the steady-state zero-frequency components excited by the double-step output of the compensator. DC vectors.

clockwise with increasing time. When the first phasor has rotated 180°, the second component is introduced to cancel the first.

The dc components of the response are shown in Fig. 3. However, the dc phasors do not rotate in time and, therefore, the steady-state output is given by the sum of the two dc phasor lengths.

Application to Systems

Fig. 4(a) shows an oscillatory negative feedback system. Fig. 4(b) shows the compensator which is placed ahead of the system input. K_0 is k in our previous equations. K_1 is $(1-k)$. Fig. 4(c) is the step response of the compensator and system combination.

The derivative of the step response shown in Fig. 4(c) is the impulse response of the combination of compensator and resonant component.

This system is linear, and its transient response to any arbitrary input is the convolution of the derivative of the response shown in Fig. 4 and the input function. For all possible inputs, the output arrives at its final value tangentially in a time $T/2$ after the input function stops varying and remains constant thereafter at its final value.

The transference of (1) was realized by using a transmission line, two potentiometers, and a summing junction, as shown in Fig. 4(b). Other methods of using a transmission line or its low-frequency analog can be used, but the method shown here is especially applicable to analog computers.

The two-pole system is the simplest case for which compensation is necessary to remove a resonant peak and, therefore, was the first case treated. The system constructed was that shown in Fig. 4(a), consisting of

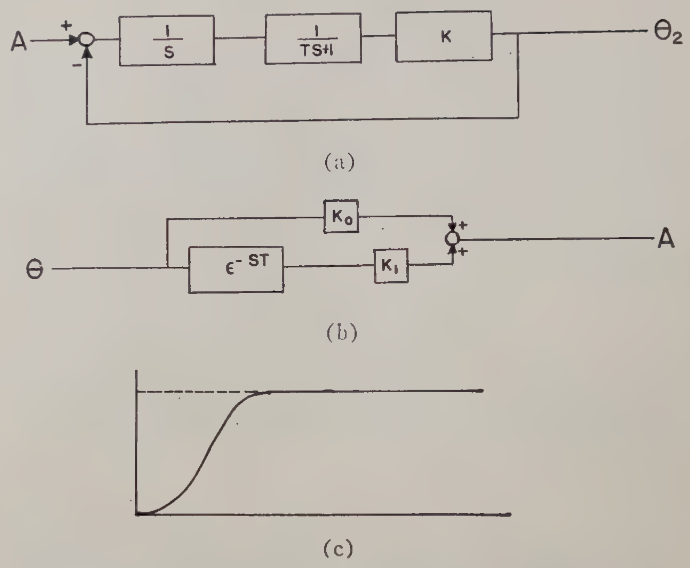


Fig. 4—(a) Two-pole system, (b) two-pole system compensator, (c) output for step input.

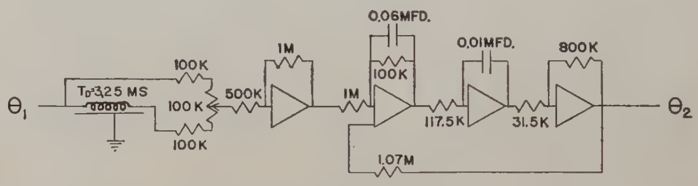


Fig. 5—Open-loop control.

negative feedback around a cascade of a dc gain, a time constant, and an integrator. This gave an open-loop s-plane configuration of two poles and no finite zeros. Our hypothesis was that the use of the compensator in Fig. 4(b) ahead of the system in Fig. 4(a) would result in a step response for the entire system like that calculated and plotted in Fig. 4(c).

Computer Results

An artificial delay line was constructed by a cascade of constant k sections, having a total delay of approximately 3.25 msec. There have been many articles recently on better approximations to a distributed parameter delay line,[2–4] but since the system considered was a low-pass filter, small deviations from a pure distortionless delay were not noticed in the output. The system which was constructed on the computer to match the block diagram of Fig. 4(a) is shown in Fig. 5. The oscillation frequency could be varied from 40 cps to approximately 1000 cps.

The frequency was set at 154 cps with the values shown in Fig. 5. If the frequency of oscillation of the

[2] S. Darlington, "Network synthesis using Tchebycheff polynomial series," Bell Telephone Sys. Mono. no. 2003; July, 1952.

[3] L. Storch, "Synthesis of constant time delay ladder networks using Bessel polynomials," Proc. IRE, vol. 42, pp. 1666–1675; November, 1954.

[4] E. S. Kuh, "Synthesis of lumped parameter delay lines," 1957 IRE Convention Record, pt. 2, pp. 160–174.

analog simulation in Fig. 5 is computed from element values, it should be 92 cps. However, the capacitors used in the integrators were of a common paper and foil type, whose nominal values were not checked and probably did not have the values shown. It was assumed that the individual computing amplifier gains were each infinite. The actual gains were not measured. The frequency of 154 cps was stable and easily reproducible over a period of several months.

The dimensionless damping, ζ, was set at approximately 0.1 to give a highly oscillatory transient. A smaller damping ratio would have been desirable, but it was found that with too small a damping ratio the pole locations would drift, sometimes into the right half s plane during the operating time. This was probably due to slow changes in amplifier gain as a function of temperature. With this circuit, adjustment of either the time constant or the dc gain varied both the frequency and damping ratio.

The compensator is shown in Fig. 5 to consist of the delay line with two inputs to a summing amplifier. The damping ratio of the compensator zeros is controlled by varying the input potentiometer. The two 100-kΩ resistors in series with the potentiometer arms were added in order to get finer control on the adjustment of the damping ratio.

The step response of the uncompensated system is shown in Fig. 6(a). The step response of the compensated system is shown in Fig. 6(b). The effect of the compensator is spectacularly apparent. Notice that the transient is completely cancelled in less than one cycle of oscillation. There is only one discrepancy in the response which would not be predicted by the above analysis. There is a slight discontinuity in slope at about three-quarters amplitude up the leading edge of the output. The reason for this is shown in the step response for the delay line alone in Fig. 6(c).

Notice that there is also a slight additional roughness in the line response alone after the delay time. This roughness could introduce additional transients into the system so that the transient sum is not equal to zero after the last excitation. By slightly altering the ratio of the line delay to the system half period, however, the sum of the transient components can be made exactly zero as shown in Fig. 6(b).

The effect of a slight misalignment of the zeros is shown in Fig. 6(d) and 6(e). In Fig. 6(d) the resonant frequency, or ω coordinate of the zeros, is reduced so that they move together along the straight line directly between the poles. The result is an increase in the residual oscillation, the magnitude being proportional to the s-plane distance between the zero and the pole. The wave is a negative sine wave after the second pulse. The phase of the error is $+90°$, due to the angle contribution of the zero nearest the pole. This was achieved by making the timing between the pulses too great, so that the second pulse arrived too late.

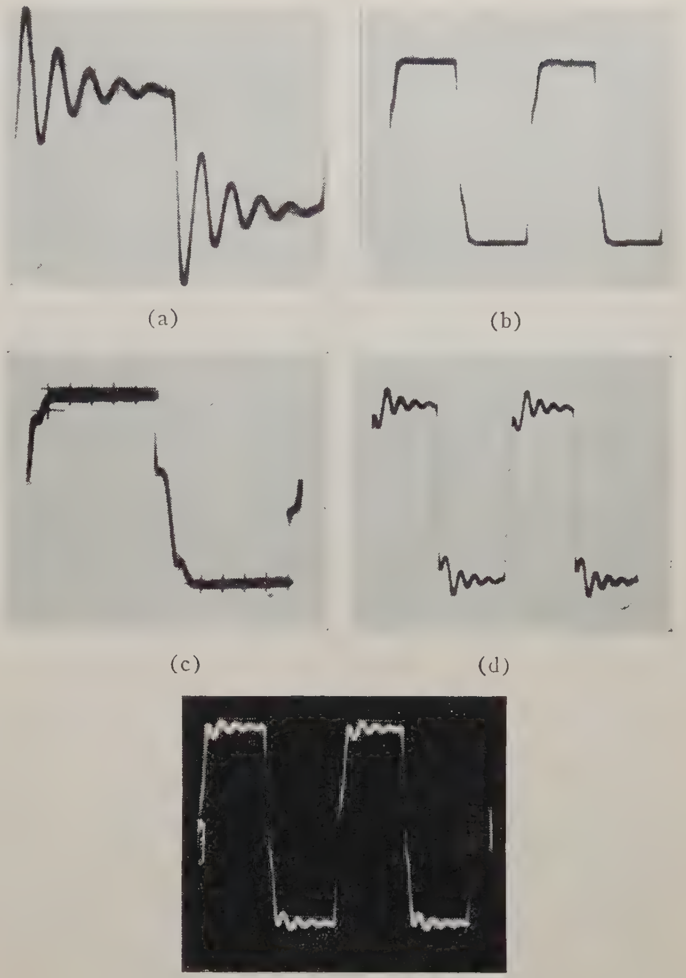


Fig. 6—(a) Uncompensated system step response, (b) compensated system step response, (c) delay line step response, (d) effect of incorrect adjustment of the compensator. The zeros are at the correct damping but too low a frequency. The transmission line was too long. (e) Effect of incorrect adjustment of the compensator. The zeros are at the correct frequency but at too great a damping. The transmission line delayed step was too small. The zero misalignment in this case is 90° from the misalignment in (d).

In Fig. 6(e) the zeros were adjusted to the correct frequency but were shifted along a line of constant frequency in the s plane to too great a value of damping. The zeros were to the left of the poles. The result is again an error oscillation. The magnitude was proportional to the distance between the zero and the pole. The wave was a positive cosine wave after the second pulse. The phase of the error was zero degrees, which was the angle contribution of the zero nearest the pole. This was achieved by making the magnitude of the second pulse too small.

Feedback Control

It is evident from Fig. 4 that the method outlined above is valid for disturbances occurring at the input only and passing through the compensator. If a small disturbance occurs at any other point in the system the system will oscillate as if there were no compensa-

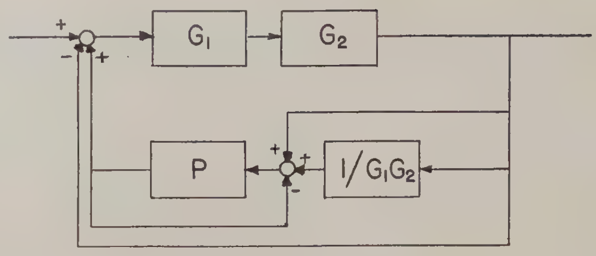
tion. To remedy this situation, the delay line can be imbedded into the feedback loop so as to correct for disturbances anywhere in the forward channel. The result of this process yields the block diagram of Fig. 7.[5,6] The pulse generator, P, which is shown in the diagram, is derived from (1) by subtracting a unit step at the origin. The transfer function of the pulse generator is then:

$$P = G - 1 = -(1 - k)(1 - \epsilon^{-sT/2}). \qquad (12)$$

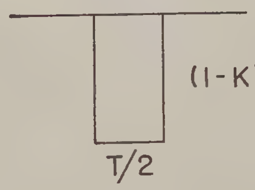
This generator is easily realized by the same network as that used in Fig. 4 for open-loop control, except that the generator delivers a step output of $-[1-k]$ initially and then follows this with a step of height $+[1-k]$ after $T/2$ seconds. The function P has no steady-state gain.

In order to realize the closed-loop control, it is necessary to generate an approximation to the inverse of the open-loop function. Since the open-loop function contains two poles, one at the origin, the inverse will contain two zeros, one at the origin. Since physical realizability dictates that this network will have some poles, however, an extra zero is introduced. This zero is made variable so that it can be used to cancel the effect of the poles at the operating frequency. The basic requirement of the network is that it have 180° phase shift at the operating frequency. The realization is shown in Fig. 8 to consist of two zero-producing sections consisting of shunt RC sections followed by a series capacitance to provide a zero at the origin. It is desirable to place the poles of this network as far out from the origin as possible in order to make the phase lag of the poles negligible at the operating frequency. As the poles are shifted outward, however, the gain of the circuit goes down, and so amplifiers are required to make up for the attenuation. The poles were placed as far out as possible while keeping the output signal above the noise level. The unit was then heavily shielded to reduce pickup.

The actual computer wiring is shown in Fig. 9, opposite. The system was the same as that used for the previously described tests. The circuit shows the additional elements required in this case added as a minor feedback loop consisting of the network for generating $1/G_1G_2$ followed by the pulse generator with positive feedback. This system was also adjusted for a dead-beat response. These adjustments were made in three steps. First, the system model was adjusted by inserting a vacuum tube voltmeter at the input to the pulse generator or the output of the adder from the feedback and the network for the model of the reciprocal function. The surplus zero position was adjusted until there was a null or minimum of transmission at 154 cps. Next, the gain of the branch paralleling the model of the reciprocal system was adjusted so the magnitude of the null corresponded to a complex zero whose Q was essentially equal to the closed loop Q of the uncompensated system. Lastly, the voltmeter was removed and the system excited with a square wave of about 20 cps. The feedback resistor around the output amplifier of the pulse generator was then adjusted to minimize the transient. The transient was not eliminated entirely but was significantly reduced in amplitude. The error was probably the result of the combination of the error in the delay line and the error in the network performing the inverse of the open-loop function. As in the previous case, this error was not important because the ratio of the transmission line delay time to the system period could be adjusted so that all transient inputs would have a dead-beat response.



(a)



(b)

Fig. 7—Closed-loop control.

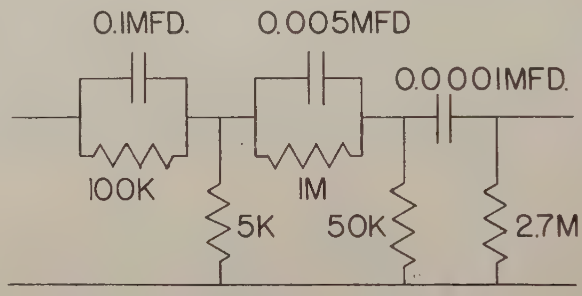


Fig. 8—Simulation of $1/G_1G_2$.

More Complex Systems

In Fig. 1 there are drawings of two different step responses of posicast compensators. The second step response defines the compensator which should be used for a real pole. If the transcendental equation of (1) is now examined for poles and zeros with $k > 1$, it is found that the poles have the same location as before, but the value of n in (7)–(9) is even instead of odd. The zeros have the same value for α but are located at frequencies of $n(2\pi/T)$ where n is even. There is one zero on the real axis of the s plane and a column of complex zeros.

[5] See Smith, *op. cit.*, for more details.
[6] O. M. J. Smith, "Mixed distributed and lumped parameter systems," *1957 IRE WESCON Convention Record*, pt. 2, vol. 1, pp. 122–132.

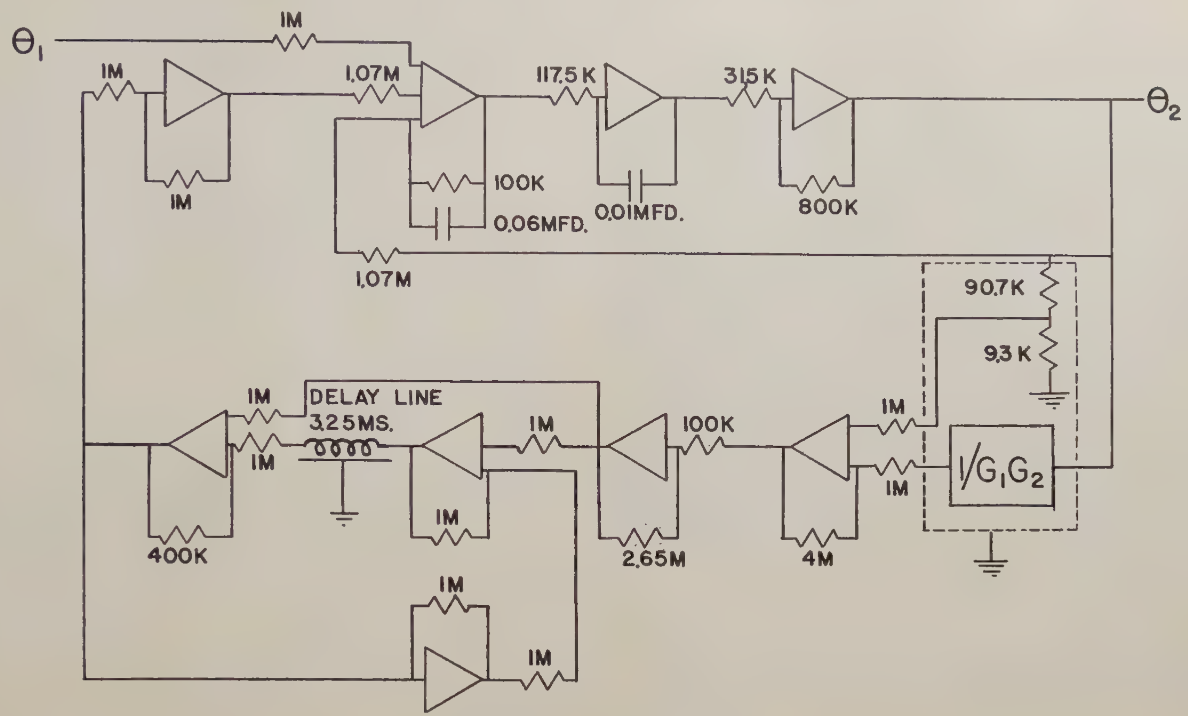


Fig. 9—Closed-loop control.

Notice that it takes only one degree of freedom to determine the real zero, and the other degree, the time delay, might be used for other poles. In other words, the time of cancellation for a real pole can be anything desired within the limits of physical realizability and the maximum corrective signal.

The method of compensating more complex systems is now clear. We assemble a compensator for each pole, or complex pair of poles, and cascade the entire system, giving a transference:

$$H = \prod_{i=1}^{m} (k_i + [1 - k_i]e^{-sT_i}). \tag{13}$$

A simpler form to realize becomes evident, however, if we convolve each of the above transfer functions together:

$$H = \sum_{i=1}^{n} \beta_i e^{-sT_i}. \tag{14}$$

where a T_i is required for each pair of resonant poles and for each real pole. This function can be realized by the same method that has been used thus far, *i.e.*, by taking taps off a delay line at the T_i and applying these points, multiplied by the proper constants, to a summing junction. For two sets of complex poles, (14) simplifies down to four terms:

$$H = \beta_1 + \beta_2 e^{-sT_1} + \beta_3 e^{-sT_2} + \beta_4 e^{-s[T_1+T_2]}. \tag{15}$$

The zeros in this expression seem to go through an interesting sequence in which they are printed into the s plane at the time T_1 to compensate for the highest frequency poles, removed from these poles at time T_2, and finally applied again at time T_1+T_2. At this last time the zeros are on top of both sets of poles. Since a real pole compensation does not depend on the time involved, the above expression will simplify even further for three-pole systems.

Faster Response Times

The above analysis is based entirely on the desirability of one-half-cycle response. The control method is general, however, and easily affords transient times of less than one-half cycle. To see the method for this type of response, we make the substitutions:

$$z = e^{sT_r} \tag{16}$$

where T_r is the time spacing between the two pulses closest together in the compensator and is very much less than the period T of the uncompensated system.

$$T_r \ll T. \tag{17}$$

This transforms the $j\omega$ axis of the s plane into the unit circle in the z plane with other lines of constant α being mapped into concentric circles about the origin. The zeros of the delay line compensator are now easily plotted onto the z plane. For the half-cycle response when $T_r = T/2$, the z-plane configuration consists of one zero on the negative real axis. The position of the zeros in (1) is determined by the following:

$$z = -[1 - k]/k. \tag{18}$$

If T_r is set equal to $T/4$, there is a change of a factor of two in the base for the z plane (basing the plane on one-quarter cycle rather than one-half cycle) and the zeros of (1) will be on the imaginary axis of the z plane.

$$z = \pm j\sqrt{(1-k)/k}. \qquad (19)$$

If transient times of less than one-half cycle are desired, it is necessary to move the zeros on a circular arc towards the positive z axis. By this means, since the frequency of the lowest zero is fixed by the system to be compensated, the next zeros in the column in the s plane are moved to a much higher frequency. This results in much greater bandwidth, and consequently shorter transient times. The transient time is $2T_r$. Note that the base of the z plane must shift with the zeros so that the radian frequency of the lowest order s-plane zero will remain the same at ω of $2\pi/T$. The perunit system transient time of $2\pi T_r/T$ in radians is given by the total included angle between the zeros in the z plane. This angle, which is shown as 2θ in Fig. 10, is equal to the number of degrees of electrical oscillation of the entire system for a step or impulse input.

$$\theta = 2\pi T_r/T. \qquad (20)$$

The transfer function of the compensator for these response times can be computed from the z plane to be:

$$H = [\beta_1 z^2 + \beta_2 z + \beta_3]z^{-2}. \qquad (21)$$

The zeros of the quadratic expression are then found and the corresponding coefficients are readily evaluated.

For no damping, the zeros lie on the unit circle in the z plane, and the coefficients in (21) are:

$$\beta_1 = \beta_3 = 1/[2 - 2\cos\theta] \qquad (22)$$

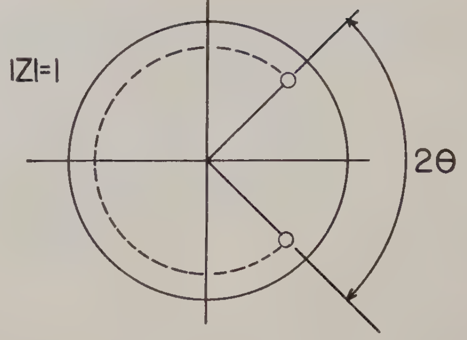
$$\beta = -[\cos\theta]/[1 - \cos\theta]. \qquad (23)$$

θ is the half-transient time in radians of the uncompensated oscillation. An example of this method is given by the coefficients for one-third cycle-response:

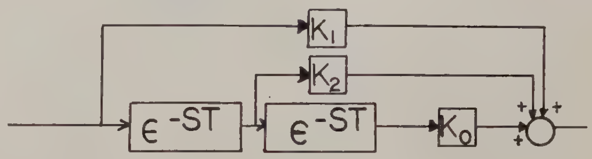
$$H = 1 - e^{-sT/6} + e^{-sT/3}, \qquad (24)$$

where T equals the uncompensated oscillation period.

As the zeros are moved closer to the real axis, the trend of the coefficients is to get larger as the axis is approached. Response times less than one-sixth cycle are not feasible unless unusual power capabilities are present in the system. In the limit, the step response of the compensator consists of a unit doublet and a unit step, both at the origin of time. The method of realization of the transfer function of the compensator for faster response times than a half cycle is shown in Fig. 10. This compensator is also an extension upon the basic compensator developed above.



(a)



(b)

Fig. 10—(a) Fractional cycle transient zeros, (b) complex zero generator.

Effect of Misalignment

The above derivations specify that the pole is exactly cancelled by the zero. For some special purposes it is advisable to introduce a slight misalignment. For example, a slightly greater bandwidth can be obtained by moving the zero up and to the left of the pole. This increase in bandwidth is bought at the expense of a slight overshoot in the step response.

For sinusoidal operation at the transient resonant frequency, the system sinusoidal gain when the zero lies exactly on top of the pole is approximately $\pi/4$. The exact gain, when the zero has the same ω value as the pole but has a different α value, is

$$\text{Gain at } \omega_n = \frac{\pi}{4}\left[\frac{1 - \epsilon^{-\pi\zeta_n(1+\beta)}}{\pi\zeta_n\sqrt{1 - (\zeta_n/2)^2}}\right]. \qquad (25)$$

ζ_n is the natural transient dimensionless damping ratio of the pole of α/ω_n or tan $\sin^{-1}(\alpha/\omega_0)$. ζ is the damping ratio based on the s plane radial distance ω_0. ζ is α/ω_0 or sin $\tan^{-1}\zeta_n$. From these relationships it can be shown that

$$\frac{\zeta}{\zeta_n} = \frac{\omega_n}{\omega_0} = \sqrt{1 - \zeta^2}. \qquad (26)$$

β is the ratio of the α value of the zero divided by the α value of the pole. It is the perunit error in the value of either α or ζ_n for the zero, measured positively to the left in the s plane.

For small values of β and of ζ_n, an approximate expression for the system gain at the transient frequency is

$$\text{Gain at } \omega_n \cong \frac{\pi}{4}\left[\frac{1 + \beta(1 - \pi\zeta_n) - \frac{1}{2}\pi\zeta_n}{1 - \zeta^2}\right], \qquad (27)$$

or very approximately,

$$\text{Gain} \cong \frac{\pi}{4}[1 + \beta]. \qquad (28)$$

A good approximation for the maximum sinusoidal resonance for a wide range of values of β is

$$M_{\max} = \frac{1}{\left(\zeta_n + \dfrac{2}{\pi(1+\beta)}\right)2(1-\zeta)^2}. \qquad (29)$$

Either (25) or (29) can be used to solve for the desired value of β, given the maximum resonant rise, M_{max}. This gives the amount of zero-location shift from the best transient response (dead-beat) as a function of bandwidth specifications.

In addition to desirable misalignments, there can be undesirable drifts in the relative positions of the zeros and the poles. Closed-loop feedback systems that contain machinery or nonlinear devices may have poles whose positions move as a function of signal level. Amplifiers and other electronic components may change gain and transfer functions with temperature changes. The net result of these effects can be represented as a pole motion, and (25), (28), or (29) give the maximum change in M at the sinusoidal resonant frequency as a function of the perunit damping misalignment β of the zero with respect to the pole.

Conclusion

A method of control has been developed which brings transients to their steady-state values tangentially in a small fraction of the normal oscillatory period by cancelling the s-plane resonant poles with z-plane zeros. The method gives excellent waveform. The frequency response of the compensated system is down only 3 db at a frequency equal to $\frac{1}{2}T$, where T is the arbitrarily chosen transient time. The method is easy to apply, the only difficult hardware being an artificial transmission line.

This mode of operation can be applied to a system with an arbitrary number of poles by a simple extension of the two-pole case which has been discussed here. The steady-state output can be obtained in as short a time as desired, the only restrictions being the dynamic range and snr of the system.

On Closed-Form Expressions for Mean Squares in Discrete-Continuous Systems*

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Summary—**When a system is to be optimized with respect to the mean square of some variable, a closed-form expression for that mean square is usually desired. The problem of obtaining such expressions for discrete-continuous systems—*i.e.*, systems made up of both sampled-data and continuous subsystems—has been a difficulty in the past. The reason for this is that the spectral densities of the variables of interest often contain rational functions of exp $(j2\pi fT)$ combined multiplicatively with rational functions of f, f being the frequency coordinate of the spectral densities, and T the sampling period. Presented here is a technique for finding the desired closed-form expressions. It is based on the relation**

$$\int_{-j\infty}^{j\infty} P^*(e^{sT})Q(s)ds = \oint P^*(z)Q^*(z)z^{-1}dz,$$

where $Q^*(z)$ is the "Z-transform" of $Q(s)$,

To illustrate the technique, closed-form formulas for the output and ripple of discrete-continuous systems and for the control error of sampled-data feedback systems are derived, and an application to a "track-while-scan" system is given.

I. Introduction

Many "sampled-data" systems in the fields of communication and control are "discrete-continuous"; *i.e.*, they are combinations of both purely "discrete" and purely "continuous" subsystems, the terms "discrete" and "continuous" referring to time-domain characteristics of the signals involved. To evaluate the performance of such systems, it is often desired to compute the mean square "system error"—*i.e.*, the average of the squared difference between the *desired* output and the *actual* output—in response to a random input.

Furthermore, it often is desired to optimize the system with respect to one or more dynamic parameters. To do this conveniently, a closed-form expression for the mean square error in terms of the dynamic parameters of the system is required. In addition, closed-form expressions for other signals in the system such as the output, or related functions such as the ripple may be needed.[1]

In the past, these mean squares have been expressed in integral forms requiring numerical methods for their evaluation.[1–4] When closed-form expressions were required, they were obtained by approximating the discrete-continuous system by an all-discrete or an all-

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[1] J. Sklansky and J. R. Ragazzini, "Analysis of errors in sampled-data feedback systems," *Trans. AIEE*, vol. 74, pt. II, pp. 65–71; May, 1955.
[2] J. R. Ragazzini and L. A. Zadeh, "Analysis of sampled-data systems," *Trans. AIEE*, vol. 71, pt. II, pp. 225–234; November, 1952.
[3] M. Mori, "Statistical treatment of sampled-data control systems for actual random inputs," *Trans. ASME*, vol. 80, pp. 444–456; February, 1958.
[4] R. M. Stewart, "Statistical design and evaluation of filters for the restoration of sampled data," Proc. IRE, vol. 44, pp. 253–257; February, 1956.

continuous system. Thus, a need for closed-form, analytical evaluations without approximations has been apparent. This paper answers this need for a class of situations frequently encountered in practice.

Given in Section II is a formula for the mean square output for systems in which: 1) the system transfer function can be expressed as a product of a rational, physically realizable transfer function of the Laplace operator s and a rational, physically realizable transfer function of e^{sT} (or a sum of such products), 2) the input's spectral density is a known rational function of frequency, and 3) the input signal is independent of the sampling process. In addition, of course, the over-all system must be stable, since otherwise the mean squares would be infinite, and the computed values of the mean squares, if finite, would be incorrect.

The techniques used in deriving this formula can also be applied to finding closed-form expressions for other time functions related to the system, such as the system error, the control error,[5] and the ripple. These techniques are described in Appendix II. Formulas for the control error and the ripple for error-sampled feedback control systems are developed in Appendix III, making use of the techniques of Appendix II.

At this point an equation displaying the essential mathematical contribution of this paper is presented. This contribution amounts to a method of evaluating the integral over all real frequencies of a product of two spectral densities, one of which is rational in e^{sT} and the other rational in s. This integral is the left member of the equation, given below, and the method of evaluating it is indicated by the right member. The basis for this relationship is derived in Appendix II.

$$\frac{1}{2\pi j}\int_{-j\infty}^{j\infty} P^*(e^{sT})\cdot Q(s)ds = \frac{1}{2\pi j}\oint_{\text{unit circle}} P^*(z)Q^*(z)z^{-1}dz, \quad (1)$$

where

$$Q^*(z) \triangleq Q^*(e^{sT}) \triangleq \mathcal{Z}Q(s) \triangleq \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} \frac{Q(p)}{1-e^{-(s-p)T}}dp. \quad (1a)^6$$

It is assumed in (1a) that c is chosen large enough to cause the line of integration, $c-j\infty$ to $c+j\infty$, to be to the right of all the poles of $Q(p)$.

II. An Analytically-Evaluable Formula for the Mean Square Output

The discrete-continuous system illustrated in Fig. 1 is considered. The input is a stationary random signal, $u(t)$. This is sampled at the rate $1/T$ to produce the train of impulses

$$u^*(t) \triangleq \sum_{n=-\infty}^{\infty} u(nT)\delta(t-nT). \quad (2)$$

A linear system, whose transfer function is the product of $A^*(e^{sT})$ and $H(s)$, responds to this "sampled" input to produce the output $v(t)$. It is assumed that $A^*(e^{sT})$ and $H(s)$ are both physically realizable transfer functions, that the former is rational in e^{sT}, and that the latter is rational in s.[7] In addition, it is assumed that $v(t)$ remains bounded for all t. This latter assumption will hold good when $A^*(z)$ and $H(s)$ are "stable," *i.e.*, when the poles of $A^*(z)$ lie inside the unit circle of the z plane and the poles of $H(s)$ lie inside the left half of the s plane.[8]

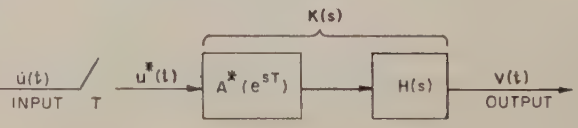

Fig. 1—A common type of discrete-continuous system.

Under these assumptions, $u(t)$ and $v(t)$ are related by

$$V(s) = U^*(e^{sT})A^*(e^{sT})H(s), \quad (3)$$

where $U^*(e^{sT})$ is the z transform of $u(t)$. ("z transform" is just a special designation for the Laplace transform of (2), with z replacing the variable e^{sT}.) Thus

$$U^*(z) \triangleq \sum_{n=-\infty}^{\infty} u(nT)z^{-n}. \quad (4)$$

Techniques of handling transforms of this type are discussed.[1,9] The latter reference is of particular interest here, because it includes some discussion of bilateral z transforms, which are associated with the autocorrelation functions of sampled random signals.

Assuming that the input's autocorrelation function, $\psi_u(\tau)$, is known, and that the corresponding spectral density, $\Psi_u(j\omega)$, is a rational function of ω, the following equation is an analytically evaluable formula for the mean square output of the system in Fig. 1. [For convenience we use the symbolic form $\Psi(j\omega)$ to denote spectral density rather than the more usual notations $\Psi(\omega)$ or $\Psi(\omega^2)$.]

$$\overline{v^2(t)} = \frac{1}{2\pi j}\oint_{\text{unit circle}} \Psi_u^*(z)A^*(z)A^*(z^{-1})\Psi_h^*(z)\frac{dz}{Tz}, \quad (5)^{10}$$

[5] A distinction is made here between the *system error* and the *control error*. The former is the difference between the desired output and the actual output; the latter is the difference between the input of a feedback system and the feedback signal.

[6] "$\triangleq$" means "equals by definition." Derivation of (1a) given by E. I. Jury, "Analysis and synthesis of sampled-data control systems," *Trans. AIEE*, vol. 73, pt. I, pp. 332–346; September, 1954.

[7] Convenient, necessary, and sufficient conditions for this assumption are given in Appendix I.

[8] J. G. Truxal, "Automatic Feedback Control System Synthesis," McGraw-Hill Book Co., Inc., New York, N. Y.; 1955.

[9] W. K. Linvill, "System theory as an extension of circuit theory," IRE Trans. on Circuit Theory, vol. 3, pp. 217–223; December, 1956.

[10] The fraction $dz/(Tz)$ is written as shown, because $dz/(Tz) = ds$, thus exhibiting the equivalence of (5) to the integral of a spectral density over the real frequency axis.

where

$$\Psi_u^*(z) \stackrel{\Delta}{=} \sum_{n=-\infty}^{\infty} \psi_u(nT)z^{-n}, \qquad (6)$$

$$\Psi_h^*(z) \stackrel{\Delta}{=} \sum_{n=-\infty}^{\infty} \psi_h(nT)z^{-n}. \qquad (7)$$

The function $\psi_h(nT)$ is defined as the autocorrelation function of $\mathfrak{L}^{-1} H(s)$, with the correlation interval equal to nT. Thus,

$$\psi_h(\tau) \stackrel{\Delta}{=} \int_{-\infty}^{\infty} h(t)h(t+\tau)dt, \qquad (8)$$

where $h(t)$ is the inverse Laplace transform of $H(s)$, *i.e.*, $h(t)$ is the impulsive response of the continuous portion of the system of Fig. 1.

The derivation of (5) is given in Appendix II. Techniques of evaluating it analytically are discussed in Section III.

Although (5) explicitly pertains to the output, the techniques involved in its derivation can be used for deriving closed-form expressions for other mean squares related to the system, such as the system error, control error, and ripple.[1,3] To illustrate this fact, analytically evaluable formulas for the mean square control error and the mean square ripple in error-sampled feedback systems are derived in Appendix III.

III. Evaluating the Formula

To evaluate (5), the closed-form expressions for $\Psi_u^*(z)$ and $\Psi_h^*(z)$ are found. Then these expressions are combined with $A^*(z)$ and $A^*(z^{-1})$ to obtain the integrand of (5), and the residues of this integrand inside the unit circle are obtained.

To find $\Psi_u^*(z)$, use either (6) in conjunction with a table such as Barker's[11] or use the relation

$$\Psi_u^*(z) = \mathrm{Z}\Psi_u(s), \qquad (9)$$

where

$$\Psi_u(s) \stackrel{\Delta}{=} \Psi_u(j\omega)\big|_{\omega^2=-s^2} \qquad (10)$$

and where Z represents the z transformation of the time function upon whose Fourier or Laplace transform Z operates, *i.e.*, Z represents a mapping from the s to the z domains. A good table of such mappings is given by Barker;[11] shorter tables are given by Truxal[2] and Jury.[6] An integral representation of the Z transformation is given by (1a).

To illustrate the method of using the tables to find $\Psi^*(z)$, consider the case where the spectral density of $u(t)$ is $a/(\omega^2+a^2)$. Replacing ω^2 by $-s^2$, $\Psi_u(s) = a/(-s^2+a^2)$. The Z transform of this expression may be found in a table or from (1a), where

$$Q(p) = \frac{-a}{(p-a)(p+a)} \cdot$$

The obtained transform is

$$\Psi_u^*(z) = \frac{-z \sinh aT}{z^2 - 2z\cosh aT + 1} \cdot$$

(Even though the tables list only the transforms of one-sided time functions, the tables can be used for two-sided time functions when going from the s to z or from the z to s domains. Essentially, this is because both the negative and positive sides of any complex exponential of the form $Kt^m e^{(\alpha+j\beta)t}$—which is the type of time-function component contributing partial fractions to the s and z transforms—will have the same s transform, and both sides will also have the same z transform.)

$\Psi_h^*(z)$ may be found in a similar manner by using (7) or the relation:

$$\Psi_h^*(z) = \mathrm{Z}[H(s)H(-s)]. \qquad (11)$$

This relation is a consequence of (7) and the fact, implied by (8), is that the Fourier transform of $\psi_h(\tau)$ is $H(s)\,H(-s)$, with s replacing $j\omega$.

To illustrate, suppose $H(s)=1/s$. Then,

$$\Psi_h^*(z) = \mathrm{Z}\left(-\frac{1}{s^2}\right) = \frac{-Tz}{(z-1)^2} \cdot \qquad (12)$$

IV. An Alternate Expression

The following alternate formula for $\overline{v^2(t)}$ stems directly from (5) through applying in reverse order the techniques used in progressing from (32) to (34) in Appendix II. This formula is of some academic interest, because the integration is made over the imaginary axis in the s domain, as in Laplace inversion; however, it offers less computational convenience than (5) because of the inverse Z transformation required.

$$\overline{v^2(t)} = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \frac{1}{T}\Psi_K(s)ds, \qquad (13)$$

where $\Psi_K(s)$ is defined by

$$\mathrm{Z}\Psi_k(s) \stackrel{\Delta}{=} \Psi_u^*(z)A^*(z)A^*(z^{-1})\Psi_h^*(z). \qquad (14)$$

To find $\Psi_k(s)$, an inverse Z transformation of (14) is made by expanding the right member into partial fractions with the form $a_i/(1-e^{-b_iT}z^{-1})$, and assigning corresponding fractions of the form $a_i/(s+b_i)$ to $\Psi_k(s)$. (Here b_i and a_i may be real, complex, or imaginary.)

A computational advantage of this formula is that available tables[12,13] have direct applicability to its evalu-

[11] R. H. Barker, "The pulse transfer function and its application to sampling servo systems," *Proc. IEE*, pt. IV, monograph no. 43; July 15, 1952. Note: to use the tables, one would have to express $\psi_u(\tau)$ as a sum of two "one-sided" time functions, one of which is zero for negative time, and the other zero for positive time. The reason for this is that the present tables of z transforms were computed only for one-sided time functions.

[12] H. M. James, N. B. Nichols, and R. S. Philips, "Theory of Servomechanisms," M.I.T. Radiation Lab. Series, McGraw-Hill Book Co., Inc., New York, N. Y., vol. 25; 1947. (See table in appendix.)

[13] R. C. Booton, Jr., M. V. Mathews, and W. W. Seifert, "Nonlinear Servomechanisms with Random Inputs," M.I.T. Dynamic Analysis and Control Laboratory, Cambridge, Mass., Rept. No. 70; August 20, 1953. (See table in appendix.)

ation. However, the same can be said of the y-domain version of (5) obtained by the transformation $z=(1+y)/(1-y)$, and, except in the simplest cases, this transformation seems to involve less labor than the $\mathcal{Z}$ inversion of (14). Furthermore, tables can be constructed which would apply directly to (5) without a z-to-y transformation.

V. Illustrative Example

The use of (5) will be illustrated by the problem of optimizing a particular track-while-scan system.[14]

Consider the sampled-data feedback system shown in Fig. 2. This forms a subsystem handling one of the cartesian coordinates in a common type of track-while-scan system. It is desired to choose values of α and β which in some sense will provide optimum performance.

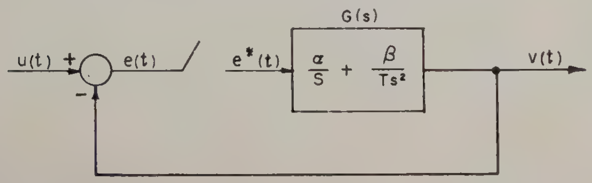

Fig. 2—The block diagram of a common type of track-while-scan system.

One of the figures of merit that has been considered for such an optimization is the ratio of the mean square of the output, $v(t)$, to the mean square of the input, $u(t)$, when $u(t)$ is "white" noise whose frequency spectrum is flat over a very wide but finite band.[15]

It can be shown that $u(t)$ and $v(t)$ are related by

$$V(s) = V^*(s)\left[\frac{1}{1+G^*(z)}\right]G(s)$$

or after making substitutions:

$$V(s) = U^*(z)\left[\frac{(z-1)^2}{z^2-(2-\alpha-\beta)z+(1-\alpha)}\right]\left(\frac{\alpha s+\beta}{s^2}\right), \qquad (15)$$

where $z=e^{sT}$, and where the time scale has been normalized so as to have $T=1$. [A note to the reader undertaking to derive (15): In computing $G^*(z)$, the discontinuity in $\mathcal{L}^{-1}G(s)$ should, for physical reasons, be replaced by a line segment whose slope is finite, although large and positive. This is equivalent to subtracting the initial value of the time function from $G^*(z)$.]

Comparing (15) with (3), we see that this system satisfies the requirements for the evaluation of $\overline{v^2(t)}$ by (5), since the bracketed factor of (15) corresponds to $A^*(z)$, and the factor to the right of the brackets corresponds to $H(s)$.

The task now at hand is to find closed-form expressions for $\Psi_u^*(z)$ and $\Psi_h^*(z)$, which are needed for insertion in (5). To do this we shall use (6) and (11).

Since the spectrum of $u(t)$ is spread uniformly over a very large though finite band, the autocorrelation function of $u(t)$ is

$$\psi_u(\tau) = \begin{cases} \overline{u^2(t)} & \text{for } \tau = 0 \\ 0 & \text{for } \tau \neq 0. \end{cases} \qquad (16)$$

Hence, by (6),

$$\Psi_u^*(z) = \overline{u^2(t)}. \qquad (17)$$

The function $H(s)\,H(-s)$ in (11) is equal to $G(s)\,G(-s)$, where $G(s)$ is defined in Fig. 2. Hence, by (11),

$$\Psi_h^*(z) = \mathcal{Z}\left[-\frac{\alpha^2}{s^2}+\frac{\beta^2}{s^4}\right]$$

$$= \frac{(\beta^2-6\alpha^2)z}{6(z-1)^4}\left[1+\frac{(4\beta^2+12\alpha^2)z}{\beta^2-6\alpha^2}+z^2\right]. \qquad (18)$$

In going from the second to the third member of (18), a table such as Barker's[11] is convenient. (An interesting alternative technique for this purpose, using "modified z transforms," is described by Mori.[3])

Substituting (17) and (18) into (5) and setting $A^*(z)$ equal to the bracketed factor of (15) obtains

$$\overline{v^2(t)} = \frac{\overline{u^2(t)}}{2\pi j}\oint_{\text{unit circle}} \frac{\beta^2-6\alpha^2+(4\beta^2+12\alpha^2)z+(\beta^2-6\alpha^2)z^2}{6[(1-\alpha)-(2-\alpha-\beta)z+z^2][1-(2-\alpha-\beta)z+(1-\alpha)z^2]}\,dz. \qquad (19)$$

This can be evaluated either by finding the residues directly, or, more conveniently, by substituting $z=(1+y)/(1-y)$ and using one of the available tables.[12,13] The result is

$$\frac{\overline{v^2(t)}}{\overline{u^2(t)}} = \frac{6\alpha^2-3\alpha\beta+6\beta-\beta^2}{3\alpha(4-2\alpha-\beta)}. \qquad (20)$$

This is a closed-form expression that can be applied to choosing optimum values of α and β. (A graphical optimization scheme making use of this expression is presented.[15])

VI. Conclusion

The techniques used in the derivation of (5), which are summarized to an important extent by (1), constitute tools for finding closed-form expressions for mean squares that previously could only be found numerically. These techniques make possible relatively straightforward optimizations of discrete-continuous systems with respect to selected mean squares. For illustrative purposes, two other formulas, namely (41) and (47), are derived in Appendix III.

[14] A track-while-scan system can be broadly defined as a system for estimating the "present" value of a time function from a sequence of "past" samples of that function.

[15] J. Sklansky, "Optimizing the dynamic parameters of a track-while-scan system," *RCA Rev.*, vol. 18, pp. 163–185; June, 1957.

The assumptions making these techniques practicable are that $\Psi_u(j\omega)$ is rational in ω; that the discrete and continuous portions of the systems are rational, physically realizable transfer functions in e^{sT} and s, respectively; and that the input is independent of the sampling process. In addition it is assumed that the over-all system is stable, so that the mean squares will not be infinite.

As an aid in evaluating the formulas obtained by these techniques, a table of integrals around the unit circle for rational integrands would be convenient. Such a table would be similar in organization to the presently available tables of integrals of rational functions over the real and imaginary axes.[12,13]

Appendix I

Physical Realizability of $A^*(e^{sT})$ and $H(s)$

To test for the physical realizability of $A^*(e^{sT})$ and $H(s)$ when these functions are rational in e^{sT} and s, respectively, it is convenient to use the following necessary and sufficient conditions.

Let $z=e^{sT}$. Assuming $A^*(z)$ and $H(s)$ are rational in z and s, respectively, they are physically realizable transfer functions if and only if they can be expressed in the forms

$$A^*(z) = \frac{a_0 + a_1 z^{-1} + \cdots + a_m z^{-m}}{1 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_n z^{-n}} \tag{21}$$

and

$$H(s) = \frac{c_0 + c_1 s + \cdots + c_p s^p}{d_0 + d_1 s + \cdots + d_{p-1} s^{p-1} + s^p},$$

where the a_i's, b_i's, c_i's, and d_i's are all real.

It will be noted that the physical requirement of finite transmission at infinite frequency [*i.e.*, $|A^*(\infty)| < \infty$ and $|H(\infty)| < \infty$] is imposed by the unity term in the denominator of $A^*(z)$ and by the equality in the degrees of the numerator and of the denominator of $H(s)$.

Appendix II

Derivation of (5)

Formulas for the spectral density of $v(t)$ is derived. This part of the derivation is a modified version of Stewart's.[4]

The spectral density of $v(t)$ is, by definition,

$$\Psi_v(j\omega) \triangleq \lim_{B\to\infty} \frac{1}{B} \left| V_B(j\omega) \right|^2, \tag{22}$$

where $V_B(j\omega)$ is the Fourier transform of $v_B(t)$, which is a section of $v(t)$ covering a time interval B units long. The subscript B will have the same connotation when used with other variables.

Referring to (3), we make the following approximation, frequently used in derivations of this kind:

$$V_B(j\omega) \cong U_B^*(e^{j\omega T}) A^*(e^{j\omega T}) H(j\omega). \tag{23}$$

The error involved in this approximation will disappear presently when B is made to approach infinity. Multiplying each side of (23) by its own conjugate results in an expression for $|V_B(j\omega)|^2$. When this is substituted in (22) we obtain

$$\Psi_v(j\omega) = \left[\lim_{B\to\infty} \frac{1}{B} \left| U_B^*(e^{j\omega T}) \right|^2 \right] \left| A^*(e^{j\omega T}) \right|^2 \left| H(j\omega) \right|^2. \tag{24}$$

Now, as shown in the literature,[1] $U_B^*(e^{j\omega T})$ can be expanded into a "Poisson sum":

$$U_B^*(e^{j\omega T}) \equiv \frac{1}{T} \sum_{n=-\infty}^{\infty} U_B(j\omega + jn\omega_0), \tag{25}$$

where ω_0 is the angular sampling frequency. Thus the bracketed factor in (23) can be expressed as a sum of spectral densities and cross spectral densities:

$$\lim_{B\to\infty} \frac{1}{B} \left| U_B^*(e^{j\omega T}) \right|^2 = \frac{1}{T^2} \sum_{m,n} \left[\lim_{B\to\infty} \frac{1}{B} U_B(j\omega + jn\omega_0) \widehat{U}_B(j\omega + jm\omega_0) \right] \tag{26}$$

where "$\widehat{\ }$" indicates the conjugate function.

A typical term in this summation is the Fourier transform of

$$\frac{1}{T^2} \overline{u(t+\tau)u(t)e^{j(n-m)\omega_0 t}}, \tag{27}$$

where the bar indicates an averaging with respect to time t. Since $u(t)$ and the sampling process are assumed to be independent, (27) is equal to the product of the time averages of $u(t+\tau)u(t)$ and $e^{j(n-m)\omega_0 t}$. The latter average is zero except when $n=m$, in which case it is unity. Hence every cross spectral density in the summation of (26) is zero. The remaining terms are the "side-band spectra" of $u^*(t)$:

$$\begin{aligned} \lim_{B\to\infty} \frac{1}{B} \left| U_B^*(e^{j\omega T}) \right|^2 &= \frac{1}{T^2} \sum_{n=-\infty}^{\infty} \left[\lim_{B\to\infty} \frac{1}{B} \left| U_B(j\omega + jn\omega_0) \right|^2 \right] \\ &= \frac{1}{T^2} \sum_{n=-\infty}^{\infty} \Psi_u(j\omega + jn\omega_0). \end{aligned} \tag{28}$$

Applying the Poisson-sum identity (25) to the right member of (28), we have

$$\begin{aligned} \lim_{B\to\infty} \frac{1}{B} \left| U_B^*(e^{j\omega T}) \right|^2 &= \frac{1}{T} \Psi_u^*(e^{j\omega T}) \\ &\equiv \frac{1}{T} \left[Z\Psi_u(s) \right]_{z=e^{j\omega T}}. \end{aligned} \tag{29}$$

where $\Psi_u(s)$ is obtained from $\Psi_u(j\omega)$ by replacing ω^2 by $-s^2$.

Putting (29) into (24), we obtain an expression for the spectral density of $v(t)$:

$$\Psi_v(j\omega) = \frac{1}{T}\Psi_u^*(e^{j\omega T}) \left| A^*(e^{j\omega T}) \right|^2 \left| H(j\omega) \right|^2. \quad (30)$$

The mean square of $v(t)$ is just the integral of (30) over all real frequencies:

$$\overline{v^2(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{T}\Psi_u^*(e^{j\omega T}) \left| A^*(e^{j\omega T}) \right|^2 \left| H(j\omega) \right|^2 d\omega. \quad (31)$$

This formula has presented an obstacle in previous investigations, because no non-numerical technique of evaluating it is apparent.

To make possible a non-numerical evaluation of (31) we express it as an infinite sum of integrals over successive frequency-domain periods of the starred functions:

$$\overline{v^2(t)} = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} \cdot \left\{ \int_{-\omega_0/2+n\omega_0}^{\omega_0/2+n\omega_0} \frac{1}{T}\Psi_u^*(e^{j\omega T}) \left| A^*(e^{j\omega T}) \right|^2 \left| H(j\omega) \right|^2 d\omega \right\}. \quad (32)$$

Making the change of variable $\omega + n\omega_0 \to \omega$ for the nth interval of integration, and remembering that the starred functions are even functions of frequency and are periodic with a period of ω_0, we have

$$\overline{v^2(t)} = \frac{1}{2\pi}\int_{-\omega_0/2}^{\omega_0/2} \left[\Psi_u^*(e^{j\omega T}) \left| A^*(e^{j\omega T}) \right|^2 \cdot \frac{1}{T}\sum_{n=-\infty}^{\infty} \left| H(j\omega + jn\omega_0) \right|^2 \right] d\omega. \quad (33)$$

Using the Poisson summation rule, (25), and making the change of variable $e^{j\omega T} \to z$, we obtain the desired result:

$$\overline{v^2(t)} = \frac{1}{2\pi j}\oint_{\text{unit circle}} \Psi_u^*(z) A^*(z) A^*(z^{-1}) \Psi_h^*(z) \frac{dz}{Tz}, \quad (34)$$

where

$$\Psi_h^*(z) \triangleq \mathcal{Z}[H(s)H(-s)]. \quad (35)$$

Eq. (7) provides an alternative to (35) for finding $\Psi_h^*(z)$. The fact that (35) and (7) are consistent can be shown by $\mathcal{Z}$-transforming the Fourier transform of (8).

Note that the relation between the right-hand members of (31) and (34) is the same as between the two members of (1). Eq. (1) has thereby been proved, and may be used hereafter in the derivations of other mean squares. Two examples of this use are given in Appendix III.

Appendix III

Mean Squares of Ripple and Control Error in Error-Sampled Feedback Systems

With the aid of (1), or, equivalently, the technique used in progressing from (31) to (34) in Appendix II, we shall obtain formulas yielding closed-form expressions for the mean square ripple and the mean square control error in feedback systems of the "error-sampled" type. The block diagram of such a system is shown in Fig. 3.

Heuristically defined, "ripple" is the difference between the output of a discrete-continuous system and some specified smoothed version of the output, as indicated in Fig. 4. (A specific definition is given.[16]) "Control error" is defined here as the difference between the input and the fedback signal, so that in a unity-feedback system it is the difference between the input and the output (see Fig. 4).

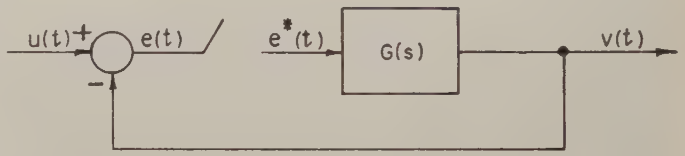

Fig. 3—An error-sampled feedback system.

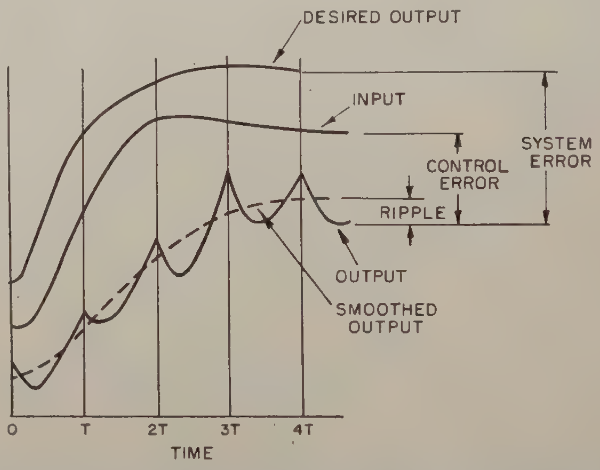

Fig. 4—Time-domain definitions of "ripple," "control error," and "system error" in a sampled-data, unity-feedback system.

Using the results of Sklansky and Ragazzini,[1] we have the following formula for the mean square ripple:

$$\overline{e_r^2(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\Psi_u(j\omega)\Psi_g^*(e^{j\omega T})}{T\left|1 + G^*(e^{j\omega T})\right|^2} d\omega - \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\Psi_u(j\omega)}{T^2}\left|\frac{G(j\omega)}{1 + G^*(e^{j\omega T})}\right| d\omega, \quad (36)$$

where

$$\Psi_g^*(e^{j\omega T}) \triangleq \mathcal{Z}[G(s)G(-s)]_{z=e^{j\omega t}}. \quad (37)$$

Again the first of these integrals can be shown to be equal to the mean square output, $\overline{v^2(t)}$. Hence, by (5), the first integral of (36) is equal to

[16] Sklansky and Ragazzini, *op. cit.*, in the section "RMS System Error," use the term "system error" as an equivalent of "ripple," because the desired output is assumed equal to the smoothed output. Awareness of this will prevent confusion.

$$\overline{v^2(t)} = \frac{1}{2\pi j}\oint_{\text{unit circle}} \frac{\Psi_u^*(z)\Psi_g^*(z)}{[1+G^*(z)][1+G^*(z^{-1})]}\cdot\frac{dz}{Tz}. \quad (38)$$

Alternatively, this expression could have been derived by applying (1) to the first integral of (36) after replacing $e^{j\omega T}$ by z.

Again applying (1), we get the following equivalent expression for the second integral of (36):

$$\frac{1}{2\pi j}\oint_{\text{unit circle}} \frac{\frac{1}{T}\Psi_u\Psi_g^*(z)}{[1+G^*(z)][1+G^*(z^{-1})]}\cdot\frac{dz}{Tz}, \quad (39)$$

where

$$\Psi_u\Psi_g^*(z) \triangleq \mathcal{Z}[\Psi_u(s)G(s)G(-s)]. \quad (40)$$

Replacing the integrals in (36) by (38) and (39), we get

$$\overline{e_r^2(t)} = \frac{1}{2\pi j}\oint_{\text{unit circle}} \frac{\Psi_u^*(z)\Psi_g^*(z) - \frac{1}{T}\Psi_u\Psi_g^*(z)}{[1+G^*(z)][1+G^*(z^{-1})]} \cdot\frac{dz}{Tz}. \quad (41)$$

This integral can be evaluated in closed-form whenever $G(s)$ and $\Psi_u(s)$ are rational functions of s.

In a similar fashion one can find an integral yielding a closed-form expression for the mean square of the control error, $e(t)$, of the system in Fig. 3. One begins with a result derived by Mori:[3]

$$\overline{e^2(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\Psi_u(j\omega) - \frac{2\Psi_u(j\omega)}{T}\,\text{Re}\left[\frac{G(j\omega)}{1+G^*(e^{j\omega T})}\right] + \frac{\Psi_g^*(e^{j\omega T})\Psi_u(j\omega)}{T\,|1+G^*(e^{j\omega T})|^2}\right\}d\omega. \quad (42)$$

The first term of the integrand contributes just the mean square input, $\overline{u^2(t)}$, to the integral. The last term contributes the mean square output, $\overline{v^2(t)}$, a closed-form evaluation for which is provided by (38). To evaluate the contribution of the middle term, we write its integral in the equivalent form,

$$-\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{T}\left[\frac{G(j\omega)\Psi_u(j\omega)}{1+G^*(e^{j\omega T})} + \frac{G(-j\omega)\Psi_u(j\omega)}{1+G^*(e^{-j\omega T})}\right]d\omega. \quad (43)$$

Since $\Psi_u(j\omega)=\Psi_u(-j\omega)$, the integral of the second term in the brackets equals that of the first. Hence (43) equals

$$-\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{2}{T}\Psi_u(j\omega)\frac{G(j\omega)}{1+G^*(e^{j\omega T})}d\omega. \quad (44)$$

Again we apply (1), this time to (44). [Reviewing the significance of (1): applying (1) to (44) is equivalent to resolving (44) into a sum of integrals over the intervals

$$\left(-\frac{\omega_0}{2}+n\omega_0,\ \frac{\omega_0}{2}+n\omega_0\right) \qquad \text{for } n = 0, \pm 1, \pm 2, \cdots$$

and making the change of variable $\omega+n\omega_0\to\omega$, followed by $e^{j\omega T}\to z$.] As a result, (44) becomes

$$-\frac{1}{2\pi j}\oint_{\text{unit circle}}\frac{2\Psi_u G^*(z)}{1+G^*(z)}\cdot\frac{dz}{Tz}, \quad (45)$$

where

$$\Psi_u G^*(z) \triangleq \mathcal{Z}[\Psi_u(s)G(s)]. \quad (46)$$

We now replace the integrals of the first, second, and third term of the integrand of (42) by $\overline{u^2(t)}$, (45), and the right member of (38), respectively. This gives us

$$\overline{e^2(t)} = \overline{u^2(t)} + \frac{1}{2\pi j}\oint_{\text{unit circle}}\left\{\frac{\Psi_u^*(z)\Psi_g^*(z)}{[1+G^*(z)][1+G^*(z^{-1})]} - \frac{2\Psi_u G^*(z)}{1+G^*(z)}\right\}\frac{dz}{Tz}, \quad (47)$$

which will yield a closed-form expression whenever $\Psi_u(s)$ and $G(s)$ are rational functions of s.

Bibliography of Sampled-Data Control Systems and Z-Transform Applications*

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Summary—The bibliography given here represents an attempt to simplify the task of searching the literature to obtain either a general acquaintance with the field of sampled-data control or to obtain information on certain specific aspects of it. Due to the close association of the Z transform and sampled-data systems, a number of papers dealing exclusively with the Z transform are included in the bibliography.

The bibliography includes only that material which in the authors' opinion represents either a significant contribution to the field or has tutorial value. The bibliography is arranged alphabetically according to the name of the author, or the first-named author in cases of coauthorship. A subject index with cross-references to the author list is also provided.

Introduction

SAMPLED-DATA systems are systems in which the data are transmitted as periodically recurring samples rather than as continuous data. The sampling action causes the output of such systems to behave in a different manner from that of otherwise equivalent continuous systems. Special techniques are required for the design of these systems and for the prediction of their performance. A sampled-data system can be linear or nonlinear, its parameters can be constant or time-varying, and the sample rate can be constant or variable.

Sampled-data problems originated with the development of automatic-tracking radar systems. The pulsed nature of radar data affects the response of the tracking servo systems. The increasing use of digital computers in control system applications has awakened further interest in the sampled-data field. This is due to the fact that digital computers can accept inputs and deliver outputs only at discrete instants of time.

The Z transform appears to provide the most direct method for the analysis and synthesis of sampled-data systems. Nearly all the recent papers in the field of sampled-data systems have utilized the Z transform, either directly or in some modified form. In addition, the Z transform has been applied to the numerical solution of continuous systems, signal flow graphs, and statistical processes.

Relatively little of the theory of sampled-data systems has thus far been published in book form. However, a very extensive number of papers are available. The bibliography given here represents an attempt to simplify the task of searching the literature to obtain either a general acquaintance with the field or to obtain information on certain specific aspects of it. Due to the close association of the Z transform and sampled-data systems, a number of papers dealing exclusively with the Z transform are included in the bibliography.

Sampled-data control systems were first studied during World War II [36, 58], in conjunction with radar tracking. The discontinuous nature of radar position data forced the designers of control systems to surrender the notions of continuous systems and to examine the effect of sampling on control system performance. Early analyses were carried out largely by means of the Laplace and Fourier transforms.

The Z-transform method was first introduced by mathematicians for solving difference equations. This method was applied to sampled-data control system analysis by Ragazzini and Zadeh [68] and Barker [4]. The work of Barker includes an analysis of transportation lag (pure delay) and a comprehensive table of transform pairs. Both Barker and Ragazzini and Zadeh discuss system analysis, prediction of response, stability by Z-transform methods, and hold circuits.

Digital compensators and analog networks can be used to improve the response of a sampled-data system. Barker [4] and Bergen and Ragazzini [9] discuss methods for digital compensator synthesis, using as criterion the desired system response (at sample instants) to a given input function, which is usually a unit step or ramp. Franklin [30] and Bertram [10] examine this technique further and utilize the mean-square error criterion for optimizing the digital compensator. An alternate approach by Jury and Schroeder [41] and Jury [44–45] uses the modified Z transform to examine behavior between sampling instants and to design the digital compensator so as to suppress "hidden" oscillations.

The mean-square error criterion is also discussed by Sklansky and Ragazzini [76] and by Blum [11].

Sampled-data systems can be analyzed by means other than Z transforms. A comparison with amplitude modulation was made by Linvill [57]. Bilinear transformations have been used by Johnson, Lindorff, and Nordling [38, 39]. An approximation method is due to Brown and Murphy [19].

Nonlinear and time-varying sampled-data systems were analyzed by Kukel [50], Kalman [47], and Friedland [32, 33]. Multiple-rate sampled servos are discussed by Kranc [49]. Multipole sampled servos were studied by Freeman [31]. The effect of finite sample width has been examined by Farmanfarma [26, 27]. Contactor servos are discussed by Chow [25].

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SUBJECT INDEX

Proposed IRE Standard Terminology for Feedback Control Systems

THE following definitions have been approved only by subcommittees. They are extensions of the previously approved IRE Standard terminology, but they are not yet IRE Standards. They are published to obtain comment and suggestions for changes before they are approved by the feedback control committee 26.0 and the IRE Standards Committee.

Should these definitions be approved as Standards? Would they be accepted as common references? Opinions will be welcomed by the editor. It is much easier to change proposed Standards than it is to change approved Standards. Comments should be made as soon as possible.

Acceleration Constant. The reciprocal of the acceleration error coefficient.

Acceleration Error Coefficient. The coefficient of the quadratic term in the power series expansion of the difference transfer ratio.

Bode Diagram. The sum of the asymptotes of the logarithms of the magnitude of the factors that constitute a transfer ratio plotted as a function of frequency.

Difference Transfer Ratio. The transfer ratio of a loop difference signal to the corresponding loop input signal.

Error Coefficient. A coefficient of the power series expansion of the difference transfer ratio, when expanded in terms of increasing positive powers of the Laplace operator.

Frequency Response. In the sinusoidal steady state, the complex ratio of the output phasor to its input phasor as a function of frequency.

Note: If the concept of frequency response is extended to nonlinear systems, the sinusoidal components being considered should be defined, as well as the conditions of test.

Gain-Crossover Frequency. Any frequency at which the logarithm of the loop gain changes sign.

Gain, Loop. The magnitude of the loop transfer ratio.

Gain Margin. The reciprocal of loop gain at a phase crossover frequency.

Note: Gain Margin is often expressed in decibels.

Log Magnitude-Phase Diagram. A plot in rectangular coordinates of the logarithm of magnitude vs phase angle of a transfer ratio, with frequency as a parameter.

Magnification. A frequency response, expressed as the ratio of the magnitude of the response at any frequency to the magnitude of a reference frequency.

Note: In low-pass systems, the reference frequency is commonly taken as zero.

Magnification, Loop. The magnitude of the return transfer ratio.

Note: This definition coincides with that of magnification for unit feedback, low-pass, single loop, system.

Note: This peak value of the loop magnification is a characteristic of the loop, indicative of loop stability.

Nyquist Diagram. A plot of the loop transfer ratio in its complex plane, corresponding to all real values of frequency, usually made for the purpose of examining the stability conditions of the loop.

Nyquist Diagram Generalized. A plot of loop transfer ratio in its complex plane corresponding to some specified contour in the complex frequency plane.

Operationally Stable. A system is operationally stable if the utilized output is under control of its input.

Overshoot, Relative. In the response of an automatic feedback control system to a step function input, the ratio of the extreme value of the output variable minus the final value, to the final value minus the initial value of the output variable, provided the final value is constant and not infinite.

Phase-Crossover Frequency. Any frequency at which the phase angle of the loop transfer ratio passes through 180°.

Phase Margin. In a feedback control loop, at any frequency, the angle equal to 180° minus the absolute value of the phase of the loop transfer ratio.

Phase Margin at Gain Crossover. The phase margin at a frequency of gain crossover.

Phase Margin, Minimum. In an absolutely stable feedback loop, the least value of phase margin for those frequencies at which loop gain is between prescribed values.

Position Constant. The reciprocal of the position error coefficient.

Position Error Coefficient. The constant term of the power series expansion of the difference transfer ratio.

Time Response. The output of an element or system as a function of time in response to a specified input. The state of the system at the time of application of the input must be specified.

Velocity Constant. The reciprocal of the velocity error coefficient.

Velocity Error Coefficient. The coefficient of the linear term in the power series expansion of the difference transfer ratio.

PGAC News

Administrative Committee Meeting

The pertinent sections of the Minutes of the Administrative Committee Meeting, held on August 20, 1957 in the Fairmont Hotel, San Francisco, Calif., are given here in the belief that PGAC members should know what the Group policies and activities are and how they are being formed. Questions and suggestions for future Group activities are always welcome. (The Administrative Committee members are listed on the inside of the front cover.)

1) Gene Grabbe reported on the organization of the PGAC.

The program for the coming IRE National Convention in New York was discussed. Gene Grabbe indicated that he would like Dave Lindorff of the University of Connecticut to serve again as program chairman for the Convention since he did an excellent job last year. It was suggested that he be aided in this work by a representative in the West, and Tom Stout accepted this assignment. Larry Cumming suggested that Dave Lindorff get further aid and Tom Mahoney accepted the assignment in the Boston area. It was agreed that Vic Azgapetian be asked to do the same in the New York area. It was felt that these four would compose an excellent program for this very important convention. The mailing list that has accumulated from previous arrangements of PGAC programs, and was used by Art Hopkin in arranging the WESCON sessions, will be sent to Dave Lindorff for his use.

The committee assignments were then enumerated by Gene Grabbe: Awards—George Biernson; Meetings—Dave Lindorff; 1957 WESCON—Art Hopkin; 1958 National Convention—Dave Lindorff; Chapters—Bob Wilcox; Papers—George Axelby; 1958 WESCON—to be assigned; Newsletter—John Ward; Publicity—to be assigned; 1959 PGAC National Convention—Louis Wadel.

The National Convention planned for the fall of 1959 was discussed to some extent. The desirability of holding it in the Dallas area and joining it to the Simulation Conference or the Southwestern IRE Conference for Region 7 was considered, but no definite decision was reached. Louis Wadel accepted the responsibility of surveying the situation, and submitting a proposal or alternate proposals. These proposals should be made available to members of the PGAC Administrative Committee in time to be reviewed before the next meeting of the PGAC Administrative Committee during the New York Convention.

2) Reports by Committees and Officers.

Papers Committee—George Axelby described the mechanism of reviewing papers. He acts as a coarse filter and rejects obviously improper material. The rest he submits to three reviewers for comments. In most cases papers are returned to the author for modification. All this illustrates the time lag between receipt of manuscript and publication. Louis Wadel suggested D. J. Simmons of Convair, Fort Worth, Tex., as reviewer.

Although advertising in the TRANSACTIONS is an approved IRE policy (at double the cost of PROCEEDINGS advertising), the committee felt that PGAC should not solicit for advertising at this time. However, it was agreed that Institute listings should be promoted. A list of organizations will be prepared by George Axelby. John Salzer suggested that a copy of the TRANSACTIONS be sent along with the requests for listing. A letter will be prepared by George Axelby and sent to Gene Grabbe and Larry Cumming for approval before being mailed out to the various organizations.

Newsletter Committee—George Axelby was asked to transmit the information to John Ward concerning this topic. It was agreed that a Newsletter is a very worthwhile project for promoting the spirit and purposes of PGAC. Numerous suggestions were made as to what the Newsletter should include. It should include actions of the administrative committee, previews of the coming TRANSACTIONS, chapter news, personal news items, etc. The Newsletter should be brief and well-written.

Secretary-Treasurer Report—John Salzer reviewed the financial and membership status of PGAC. The organization has $9,319.91 which seems an excellent state of affairs considering the brief history of PGAC. Membership now stands at a total of 2568 of which only 75 are unpaid. Gene Grabbe made a correction of this figure to 2570 by transferring to John Salzer $4 he had collected from newly recruited PGAC members.

Chapters Committee—With Bob Wilcox absent, committee members reported on the various chapters.

The formation of the San Francisco or Bay Area Chapters were affected by L. K. Lee leaving this area. Fortunately, Art Hopkin of the University of California agreed to initiate work on the San Francisco Chapter. George Axelby reported on the Baltimore Chapter which had two combination dinner and technical meetings last year attended by about 65 and 45 persons, respectively. This chapter plans five meetings in the next season, some of which will be coordinated with the local AIEE Technical Group on Feedback Control. Tom Mahoney reported on the Boston Chapter which had five meetings last year with the attendance ranging from 35 to 125. He thought the success of the meetings was closely related to the prominence of the speaker.

Louis Wadel reported on the Dallas-Fort Worth Chapter. He thought the chapter's attendance had been somewhat low, but had great hopes for future interest and growth. The present chapter chairman is D. J. Simmons.

John Salzer reported on the Los Angeles Chapter of which he had been chairman to this time. Meeting attendance ranged from a dozen to over 300. Several of the meetings had been with other groups and were purposefully held at various areas of the Los Angeles Section. The chapter has acquired a real momentum and the coming year is expected to be even more successful.

It was noted that a new chapter was formed in Philadelphia.

Consideration was given to the lack of a PGAC chapter in the Detroit area. John Salzer indicated that he might write a letter to A. C. Hall, Director of Bendix Research in Detroit, concerning chapter formation.

The Affiliate Plan—The IRE Affiliate plan was discussed. Larry Cumming noted that the mechanism for affiliate membership in the IRE is completely set up. The plan has not gained momentum yet and there are actually very few affiliate members so far.

3) Meetings.

The Nonlinear Symposium—This symposium, held in San Francisco prior to the committee meeting, attracted a registration of 114 persons plus speakers and panel members. The luncheon was attended by more than 100 persons. Cost of the convention was $75 for the meeting room and $573.75 for program printing.

AIEE October, 1957, Conference on Computers in Control, Atlantic City—Gene Grabbe reported that PGAC was a participating organization and contributed about a half dozen papers.

ASME, April, 1958, Conference on Optimizing Control, Delaware and AIEE, April, 1958, Conference on Automatic Techniques, Detroit—These are two meetings which PGAC is interested in supporting.

The National Control Conference, October, 1959—This conference has been discussed above and will be a PGAC-sponsored affair. Louis Wadel is in charge of plans.

IRE National Convention, March, 1958—This convention has also been discussed above. Dave Lindorff is in charge of the program.

4) Other Business.

Gene Grabbe announced the formation of a National Automatic Controls Committee to represent the U. S. in the International Federation of Automatic Controls. NACC is a vehicle to achieve cooperation among five American groups in the automatic control field: IRE, AIEE, ASME, AICHE, and ISA. The next meeting of the IFAC was held in Paris on September 10, 1957 with the following representation for NACC: J. Lozier for IRE, Harold Chestnut for AIEE, and Rufus Oldenburger for ASME.

To provide NACC with general working funds, the PGAC administrative committee voted a contribution of $300 which was matched by other member societies of NACC.

Mr. Petrie, of the IRE Student Relations Committee, brought up the question of getting students interested in automatic controls at the high-school level. He suggested that at a convenient time PGAC might consider appointing a committee to work out plans for achieving this purpose. One such plan might be to set up instructions improvising some simple controls that a student could build at school or in his home. Another approach might be the provision for certain competitive tasks with prizes.

Roster of PGAC Members

Listed by IRE Regions and Sections as of January 15, 1958

A year ago, the membership of the PGAC was published in the February, 1957 issue of these TRANSACTIONS. Since that time the membership has more than doubled. To indicate the growth of the PGAC and the engineers who identify themselves professionally with automatic control, the membership directory is again published by regions and section including members of foreign countries.

Region 1

Binghamton

Bernstein, R.
Bosman, E. H.
Dibb, G.
Evans, B. O.
Gibson, R. G.
Hamburgen, A.
Hemstreet, H. S.
Ivy, R. C.
Johnson, G. W.
Kercher, R. B.
Kilmer, F. G.
Kovalchick, N.
Lavender, R. W.
Lohman, I. H., Jr.
Shatz, J. R.
Sitterlee, L. J.
Van Horn, J. F.

Boston

Ackerman, S.
Aiken, R. T.
Alden, J. M.
Alexander, C. S., Jr.
Allen, J.
Anderegg, J. S.
Annis, E. W., Jr.
Antul, J. J.
Applegate, C. E.
Apsokardos, E.
Barry, J. G.
Batchelder, L.
Beaudette, C. G.
Beaumariage, D. C.
Beecher, A. E.
Bell, C. G.
Benkley, F. G.
Bennett, H. W.
Bennett, R. K.
Biernson, G. A.
Blanchard, R. L.
Bliss, Z. R.
Bloomfield, B.
Bosch, F. M.
Bradley, C. L.
Brew, R. A.
Brindley, W. E.
Brooks, P. R., Jr.
Brown, B. D.
Brown, G. S.
Brown, J. B.
Bullard, A. H., Jr.
Burwen, R. S.
Buxton, E. T., Jr.
Capen, E. B.
Carrier, G. T.
Chin, B. M.
Chu, T. H.
Clapp, C. W.
Clements, D. F.
Connelly, M. E.
Cook, C. G.
Cox, G. C.
Dandreta, W.
Davidow, W.
Demrow, R. I.
De Russo, P. M.
Dickson, A. W.
Donnelly, F. E., Jr.
Donovan, W. C.
Dratch, J. E.
Dworshak, F. G.
Edie, J. L.
Engel, J. S.
Engelberger, J. K.
Estes, S. E.
Evans, R. R.
Fallows, E. M.
Farmer, J. W.
Farnsworth, E. P. V. T.
Farrah, H. R.
Fegley, K. A.
Fertig, K.
Fitzmorris, M. J., Jr.
Frank, W. I.
Freeman, L. D.
Freudberg, R. L.
Fricks, R. E.
Galagan, S.
Gallagher, E. F.
Garber, T. A.
Gitelman, E.
Goldberg, D.
Gordon, B. M.
Goulder, M. E.
Grossimon, H. P.
Guethlen, V. J.
Gutwill, L. A.
Hamza, M. H.
Hansen, J. T.
Heaviside, M. G.
Hersh, A. I.
Heuchling, T. P.
Hillman, H. D.
Hills, F. B.
Hills, W. L.
Holubow, F. E.
Hoogasian, L.
Hopkins, A. L., Jr.
Howard, R. A.
Huffman, D. A.
Hulburt, H. J., Jr.
Iffland, J. J.
Jamgochian, E.
Johnson, E. P., Jr.
Kaiser, J. F.
Kaplan, M. L.
Karmel, P. R.
Kasvand, T.
Kaye, M. G.
Kellner, R.
Kerk, C. T.
Khambaty, M. B.
Kilroy, H. P.
Kleinrock, L.
Kodis, R. D.
Korn, S. J.
Kotliar, A.
Kremheller, W. G.
Krulee, R. L.
Lechner, R. J.
Leonard, C. E.
Leonard, R. R.
Levy, D. M.
Lincoln, A. J.
Lindsay, G. A.
Lovell, R. W.
Lucas, E. J.
Lynch, E. E.
Mahoney, T. F.
Marino, A. S.
Mark, R. B.
Markey, J. T.
Martin, D.
Martin, L. H.
Martin, R. A.
Martin, T. E.
Masi, J. D.
Max, S. M.
McMurtrie, D. L.
McNamara, J. N.
Meditch, J. S.
Melanson, F. J.
Mercer, W. R.
Merrill, B.
Miles, R. J.
Milias, E.
Miller, H. A.
Minsky, M. L.
Misek, V. A.
Morgenstern, J. C.
Morris, R. V.
Mortensen, R. E.
Murano, L.
Nagy, F., Jr.
Narud, J. A.
Naylor, T. K.
Neidorf, E.
Newton, G. C., Jr.
Nielsen, C. E., Jr.
O'Brien, D. G.
Oettinger, A. G.
Olsson, E. A.
Orenberg, A.
Osman, M. S.
Packard, R. H.
Palmer, P. J.
Pantazelos, P. G.
Parker, D. A.
Pastan, H. L.
Pease, W. M.
Perron, R. R.
Perry, B. D.
Perry, K. E.
Petteruti, A. J.
Phillips, D. R.
Platt, H. J.
Poor, V. D.
Prival, H. G.
Pugh, A. L., III
Pughe, E. W., Jr.
Reichard, R. W.
Repella, N.
Rittenburg, S. E.
Roch, M. E.
Roediger, F. E.
Rojak, F. A.
Rona, T. P.
Rowell, W. G.
Sabin, E. A.
Sabo, J. D.
Sallen, R. P.
Savage, J. E.
Scanlon, W. C.
Scheidenhelm, R.
Schochet, J. R.
Schorr-Kon, J. J.
Schramm, M. W., Jr.
Seifert, W. W.
Shansky, D.
Sheingold, D. H.
Shortell, A. V., Jr.
Shulman, L.
Sims, B. S.
Sinclair, D. B.
Small, R. M.
Smith, B. K.
Smith, R. L.
Smith, T. B.
Snyder, D. C.
Soderstrom, R. E.
Solomonoff, R. J.
Spink, P. G.
Steinberg, C. A.
Strassberg, D. D.
Susskind, A. K.
Swonger, C. W.
Taylor, H. P.
Teixeira, N. A.
Thayer, W. D.
Tsao, C. K. H.
Tunnicliffe, W. W.
Vacca, R. H.
Valpey, R. S., Jr.
Vander Velde, W. E.
Ventura, J. J.
Viggelaar, H. V. D.
Vincent, G. D.
Walsh, W. P.
Ward, J. E.
Wexler, H. T.
Whitcraft, W. A., Jr.
White, D. J.
Whittaker, H. F., Jr.
Wilcox, R. B.
Wilkie, L. E.
Williams, S. B.
Wing, J.
Wiren, J.
Woodman, R. F.
Woodruff, T. E.
Woodward, J. H.
Young, F. M.
Zieman, H. E.

Buffalo-Niagara

Aines, F. G.
Archibald, W. R.
Beneke, J.
Frank, L. E.
Gray, R. B.
Green, V. H.
Grose, C. W.
Halsted, G. P.
Hawes, E. G.
Hayman, R. A.
Johnson, A. B.
Meister, L. E.
Michaelis, T. D.
Newton, D. J., Jr.
Novo, R. B.
Powell, F. D.
Rochin, D. W.
Ruda, E. V.
Savi, L.
Walbesser, W. J.

Connecticut

Bourret, C. J.
Brockett, R. I.
Brooks, F. A., Jr.
Brownell, R. M.
Curtiss, R. H.
Delany, E. B.
Dellenbaugh, F. S.
Dolan, O. F.
Farwell, R. P.
Flynn, T. F.
Georgi, E. A., Jr.
Gilchrist, E. S.
Gouyet, J. P. J.
Gray, N. L.
Gutterman, A. Z.
Haas, V. B., Jr.
Hall, B. A.
Harrington, C. F., Jr.
Henry, H. E.
Hubbard, R. G.
Jensen, H. T.
Kidwell, N. E.
Kosowsky, L. H.
Krauss, H. G.
Lamb, J. J.
Leuze, W. F.
Lindorff, D. P.
Lutz, C. H.
Maltais, W. E.
Mapes, T. J.
Miller, E. F.
Montgomery, R. H., Jr.
Murphy, R. E., Jr.
Murray, G. J., Jr.
Narendra, K. S.
Nelson, G. E.
Northrop, R. B.
Norton, C. A.
Orell, F. M.
Palmero, A.
Passera, A. L.
Penney, A. W.
Petruchelly, V. J.
Plehaty, S. L.
Poland, W. L.
Sherman, P. M.
Sontheimer, C. G.
Stephanz, G. H.
Stockman, W. E.
Stoker, W. C.
Sunega, E. M.
Sweet, R. D.
Swift, W. C. G.
Torrance, J. H.
Townsend, F. H.
Williams, J. B., Jr.
Zweig, F.

Erie

Lempert, J.

Ithaca

Alcaide, H. D.
Criswell, T.
Dockwiller, W. R.
Hufnagel, R. E.
Jackson, A. S.
Malone, D. M.
Martin, A. R.
Mayer, H. F.
McLange, T.
Meserve, W. E.
Phillips, T. A.
Scudder, H. J., III
Todd, J. L., Jr.

Rochester

Berch, W. H.
Brown, G. A.
Chesna, J.
Dutcher, B. C.
Ellis, T. E., Jr.
Enslein, K.
Faust, A. C.
Federici, J. T.
Formicola, A. F.
Haskell, J. W.
Heit, J. C.
Johnson, R. M.
Merle, C. W.
Morse, J. E.
Rodatus, E. J.
Ruppert, J. R.
Shalloway, A. M.
Sheehan, J. F.
Shepard, W. H.
Stone, D. J., Jr.
Thalner, R. R.
Traub, R. A.
Trott, M.

Rome-Utica

Baldridge, B. H.
Glaser, G. J.
Hatfield, J. P.
Kabrisky, M. J.
Leney, H. G.
Ross, P. C.

Schenectady

Acton, E. S.
Borner, E. F.
Buchhold, T. A.
Buscher, R. G.
Chestnut, H.
Dabul, A.
Dodson, G. C., Jr.
Dull, M. J.
Fanuele, F. J.
Kirchmayer, L. K.
Liebowitz, B. H.
Lippitt, D. L.
Pester, R. F.
Rothe, F. S.
Shuey, R. L.
Stromer, P. R.
Stutt, C. A.
Wright, W. H., Jr.

Syracuse

Bessette, D. U.
Brady, D. J.
Buchta, J. C.
Cottle, D. W.
Cowan, F. C.
Edwards, K. A.
Eibeck, A. C.
Fehlau, C. E.
Grisetti, R. S.
Heartz, R. A.
Johnson, G. L.
Jureller, J. F.
Mayo, B. R.
McCarthy, J. J.
Neelands, L. J.
Russell, J. B., Jr.

Shuart, O. H.
Smith, G. H.
Stabler, E. P.
Vaughan, J. A.
Zuroff, C. A.

Western Massachusetts

Anderson, J. M.
Lovell, B. W.
Luoma, R. A.
Mahar, T. F.
Patch, R. J.

Region 2

Long Island

Adise, H. H.
Agree, I.
Amato, J. A.
Austin, G. A.
Barber, E.
Bargeski, A. J.
Beck, D. H.
Behn, E. R.
Bennett, H. A.
Boxer, R.
Buehrle, W. E., Jr.
Burgess, E. G., Jr.
Burr, R. P.
Burt, R. F.
Cap, S. T.
Cardwell, R. A.
Caruthers, F. P.
Chapman, P. W.
Chartoff, P.
Cheng, T. H.
Cohn, N. M.
Comenzo, L. F.
Corrado, V. M.
Costa, T. A.
Crosby, M. G.
Dettinger, D.
Detwiler, S. P.
Di Toro, M. J.
Dmytrasz, J. N.
Doersam, C. H., Jr.
Dyer, J. N.
Egnuss, E. M.
Ellis, P. H.
Engelson, H. R.
Feay, D. I.
Firth, L. G., Jr.
Fishbein, M.
Fonseca, A. P.
Forman, L. E.
Franceschini, J. B.
Frank, F. P. E.
Freed, A.
Freeman, H.
Fried, G.
Friedman, E. D.
Fromm, W. E.
Fullerton, W. T.
Gennard, R. D.
Gilbert, R. C.
Giordano, S.
Glixon, H. R.
Glowacki, J.
Gneses, M. I.
Gold, M. R.
Goodstein, J.
Gordon, R. L.
Gorelick, G.
Grabbe, D.
Greene, G. F.
Gretz, R. W.
Gross, S. H.
Guido, L. A.
Hall, R. L.
Hammer, A. E.
Hansen, H. R.
Harrison, S.
Haynes, N. M.
Heacock, W. J., Jr.
Herman, S.
Hittner, C.
Hoffer, L. H.
Huber, W. J.
Iernan, F. V.
Jacob, G. W.
Joline, E. S.
Julich, H.
Kay, L. M.
Kfoury, N. F.
King, L. H.
Klein, R. C.
Knocklein, H. P.
Knox, R. W.
Kokkinos, C.
Kaird, J. A., III
Lampathakis, K. E.
Laspina, C. A.
Laudon, H. C.
Lawrence, C. W.
Lee, C. H.
Lemanczyk, J. C.
Lenefsky, S.
Levenstein, H.
Levinson, E.
Lieberman, A. G.
Marcinkowski, E. R.
Marston, R. S.
Match, M. J.
McPherson, D. L., III
Meirowitz, R. L.
Menes, J.
Mesecher, M. G.
Moritz, F. G.
Murphy, G. N.
Murphy, R. B.
Murtagh, J. B.
Odessey, P. H.
Osder, S. S.
Osterndorf, J. F.
Otten, H. C.
Packer, L.
Padovano, R. J.
Papke, F. E.
Pearsall, C. H., Jr.
Perliss, R. E.
Peterson, H. O.
Pighi, L. H.
Pirone, C.
Price, D.
Prichard, J. S.
Purdy, R. L.
Redmond, K. P.
Rehberg, C. F.
Rolinski, A. J.
Rosenthal, S. A.
San Giovanni, C., Jr.
Sant Angelo, M. A.
Sayer, G.
Schimsky, D.
Schneider, R. M.
Schneiderman, W.
Schroeder, K. R.
Schulkind, D.
Scott, J. E.
Seckler, P. J. A.
Selnick, L. L.
Sherry, L. I.
Shooman, M. L.
Shulsinger, E. R.
Sieminski, E.
Silverstein, I.
Simon, R. I.
Simonelli, N. A.
Simonton, L. J.
Skwarek, F. J.
Smilowitz, S. N.
Soboleski, W. P.
Solomon, L. I.
Spence, H. W.
Stamler, L.
Steinberg, W. A.
Stephenson, J. G.
Stern, D. L.
Tamasi, A. P.
Tangredi, G. F.
Teltscher, E. S.
Torn, L. J.
Trunk, E. G.
Tucker, A. G.
Varnum, A. M.
Vinarub, M. E.
Vogel, E.
Walker, A. C.
Wallace, R. A.
Ward, H. C.
Warren, S. D.
Wathen, R. L.
Weiner, G.
Weintraub, I.
Weiss, M.
Werst, M. C.
Westover, T. A.
Whaley, W. W.
Wheeler, H. A.
Wiesner, L.
Wild, J. J.
Winzemer, A. M.
Witmer, F. S.
Witt, C. J.
Wong, N. F.
Woolf, J.
Wurman, G.
Wyle, H.
Young, V. J.
Zetkov, G. A.

New York

Albanese, A. P.
Alexandro, F. J., Jr.
Arcand, A. T.
Archbald, R. W.
Armstrong, R. W.
Bader, B.
Banner, L.
Barker, D. R.
Baum, M. C.
Baumann, D. A.
Baxter, D. W.
Beaver, M. W.
Behnke, G. H.
Bell, R. M.
Bellantoni, J. F.
Bergen, A. R.
Berk, J.
Berl, S.
Bernstein, M. H.
Bertram, J. E.
Bibbero, R. J.
Bigelow, S. C.
Blecher, F.
Bolton, A.
Boyanovitch, D.
Brailey, M. L.
Brandt, W. E.
Caligiuri, H. J.
Callais, R. T.
Caporale, J. J.
Cassidy, J. J.
Chang, S. S. L.
Clemens, G. J.
Cohen, L. B.
Cohen, S.
Cohn, M. R.
Connelly, J. J.
Cressey, J. J. J.
Cuccia, J. F.
Cypser, R. J.
D'Amato, R. J.
Defilippis, L. S. M.
Defloria, R. N.
Deutsch, S.
De Witt, R. G.
Dickstein, S. R.
Diebold, J. T.
Dolobowsky, I.
Drossman, M. M.
Duffy, J. J.
Edwards, A., Jr.
Elder, C. B.
Evin, N. I.
Fabricant, B. S.
Feinerman, B.
Fernald, O. H.
Ferrara, L. P.
Fifer, S.
Fink, S. B.
Fleisher, H.
Freudenberg, B.
Friedensohn, G.
Friedland, B.
Garofalo, V. J.
Garson, E. L.
George, R. D.
Gilman, G. W.
Gister, S.
Glantz, L. M.
Glasser, R. M.
Glazer, E.
Glomb, J. D.
Golden, D.
Goldman, N. I.
Goldsmith, A. N.
Gourgey, A. A.
Grace, M. I.
Grayson, L. P.
Groginsky, H. L.
Gronner, A.
Grossman, R.
Guardino, R. C.
Guenther, R.
Haddad, R. A.
Harries, W.
Harty, L. T.
Hauerstock, C.
Havel, J. M.
Ho, K. W.
Hoekstra, R., Jr.
Horowitz, I. M.
Jacobs, E. I.
Jorysz, A.
Kaplan, K. R.
Kaplowitz, M.
Kassel, A.
Katell, E.
Katz, M. D.
Kelley, N. D.
Kline, B. H.
Knapp, J. Z.
Koepcke, R. W.
Korrol, C. R.
Kutcher, M. M.
Kuzmyak, M. G.
Lampert, L.
Lannary, J.
Lazarus, J. P.
Liebman, P. M.
Lindner, N. J.
Liu, B.
London, F. H.
Low, F.
Luiggi, R. H.
Maedel, G. F.
Malina, M.
Marcinkowski, H. L.
Marolda, E. A.
Marshall, S. L.
Massell, E.
Meyers, S. J.
Mohr, H. F.
Molnar, R. J.
Mond, L. I.
Newman, E. G.
Novick, W.
Oppenheimer, H. N.
Ortmann, M. W.
Parker, J. A.
Paschetto, E. J.
Perry, L.
Piore, E. R.
Porter, R. W.
Prager, S.
Preiss, R. J.
Preston, F. S.
Putterman, H.
Ragazzini, J. R.
Reeves, J. F.
Reynolds, G.
Ripp, H.
Roberts, R. P.
Rosaler, R. C.
Rosenberg, A. E.
Rosenthal, L.
Roth, E. J.
Roth, J. E., Jr.
Royer, G. H.
Ruf, F. C.
Rugge, R. A.
Sakson, P. J.
Sales, M.
Sarachik, P. I.
Schischa, E.
Schoenfeld, R. L.
Schwarz, R. J.
Sherman, S.
Shinner, S. M.
Shivack, I. M.
Shively, R. R.
Siegel, R.
Sigona, J. F.
Silvera, R. V.
Sipress, J. M.
Slavin, M. J.
Soreny, E. V.
Spratt, E. L.
Steinberg, I.
Strube, A. R.
Tartanian, C. N.
Thomas, R. O.
Townsend, R. L.
Truxal, J. G.
Turchiano, M. W.
Turczyn, W. A.
Ur, H.
Vigants, A.
Vitorovich, N.
Wallace, M.
Walton, J. S. V.
Warshauer, W. J.
Watkins, J. E.
Weeks, J. T.
Weiss, A. J.
Weitman, I.
Wernick, J. I.
Willis, P. A.
Winkelstein, R. A.
Wolfson, R.
Wolin, W.
Wood, A. G.
Young, K. P.
Young, R. O.
Yu, E. K. C.
Zadeh, L. A.

Northern New Jersey

Aaron, M. R.
Abraham, R. P.
Acker, J. L.
Alexander, E. J.
Anderson, N. E.
Antonazzi, F. J.
Archidiancono, J.
Bahls, W. E.
Bailey, E. M.
Baumann, A. H.
Bearman, A. L.
Benz, B. D.
Bogner, I.
Boubli, E. J.
Bowker, M. W.
Brendle, T. A.
Brown, A. T., III
Brown, C. S.
Brown, R. I.
Bunko, M.
Cater, J. R.
Cowles, W. W.
Cynamon, J. J.
Davis, E. S.
Digrindakis, M.
Doba, S., Jr.
Doniger, J.
Dorr, R. E.
Duffy, J. J.
Earnshaw, E. F., Jr.
Elwell, H. G., Jr.
Farrar, R. L.
Fromer, M.
Gardner, J. L.
Goldberg, S. R.
Gordon, M. J.
Grandmont, P. E.
Haas, K. A.
Hamlin, J. C.
Hamming, R. W.
Horowitz, L.
Huang, R. Y.
Humphrey, R. M.
Hunter, W. B.
Hutson, D. E.
Karwas, F. C.
Kelly, J. H.
Klarman, K. J.
Krall, W. F.
Krein, J. M.
Krist, H. K.
Kuehler, H. R.
Kunkel, E. A., Jr.
Lazos, N. J.
Leeds, M.
Losher, M. I.
Lozier, J. C.
Lunney, R. E.
Malone, M.
Markosian, V.
Mathews, M. V.
Mayo, J. S.
McCrory, J. R.
McIntyre, J. W.
Meinholtz, H. J.
Metzger, J. E.
Morrow, S. R.
Mount, E.
Mueller, P. L.
Mulligan, J. H., Jr.
Murray, A. A.
Nielsen, D. M.
Ossanna, J. F., Jr.
Otis, A. N., Jr.
Panter, P. F.
Paradise, R. Y.
Pardee, S.
Podell, R. L.
Reilly, R. A.
Richards, G. P.
Robinson, A. S.
Rosenthal, M. S.
Rossi, S.
Russell, F. A.
Schnall, E.
Scully, J. F.
Seckler, H. N.
Seergy, C. M.
Shangraw, C. C.
Shapiro, O.
Shim, I. H.
Sippach, F. W., Jr.
Smith, E. J.
Stiefel, K. E.
Streeter, T. W., Jr.
Sudduth, W. B.
Sutton, R. H.
Sweeney, W. R.
Thompson, C. F.
Warden, F. W.
Wilde, A. E., Jr.
Willes, R. L.
Winter, R. A.
Wolfe, T. R.
Woodman, R. A., Jr.
Worhach, R.
Yamagami, Y.
Zayac, F. R.
Zimmerman, A. P.
Zimmerman, L.

Princeton

Beck, G. A.
Clarke, D. R.
De Versterre, W. I.
Downie, D. E.
Faustini, C.
Garretson, E. B.
Hellstrom, M. J.
Hoedemaker, R. W.
Kang, C. L.
Maitra, K. K.
Norton, J. A.
Pressey, C. W.
Rogers, A. E.
Ruble, G. B.
Schofield, C. R.
Schorr, H. I.
Sklansky, J.
Surber, W. H., Jr.
Szwedo, S. G.
Thompson, T. H.
Truitt, T. D.
Westneat, A. S., Jr.
Whitehurst, J. D.

Region 3

Atlanta

Ackerman, C. L.
Bohr, E. T.
Bradfute, G. A., Jr.
Christopher, J. P.
Daugherty, B. W.
Eckel, J. R., Jr.
Furrow, M., Jr.
Glaser, H. I.
James, C. E.
Jones, R. C., Jr.
Pippin, R. F., Jr.
Pouncey, A. D.
Pringle, J. J., III
Robertson, D. W.
Williams, B. C.
Williams, R. E.
Ziegler, N. F.

Baltimore

Axelby, G. S.
Baida, S.
Barrack, C. M.
Bastow, J. G., Jr.
Beatty, B. M.
Behm, G. T.
Bonham, R. E.
Buchan, J. F.
Burns, J. E.
Choksy, N. H.
Chicanowicz, H. J.
Dietz, J. H.
Edwards, R. L., Jr.
Esterson, G. L.
Fegely, W. D.
Gambrill, R. D.
Glaser, E. M.

Groszer, A. J., Jr.
Hauf, J. C., III
Hay, R. E.
Hayden, H. P.
Horn, R. E.
Ichniowski, F. C. J.
Jackson, J. H.
Jentilet, A.
Jones, L. G. F.
Jones, W. N.
Kegel, A. G.
Kernan, P.
Kintner, P. M.
Leahy, F. N.
Lee, M.
Lory, H. J.
Matysek, J. J.
McNew, G. D.
Meador, A. B., Jr.
Medlin, R. O., Jr.
Merchant, S. A., Jr.
Miller, W. H., Jr.
Mortimer, T. S.
Myers, F. G.
Obermayer, R. W.
Osborne, E. F.
Paskman, M.
Penabad, J., Jr.
Pincoffs, P. H.
Plath, R. H.
Randolph, G. W.
Raynes, H. D.
Rubenstein, S. E.
Rutstein, H. S.
Sander, W. E.
Stebbins, W. J.
Stefan, R. S.
Taragin, S.
Thomas, J. F.
Titen, H.
Tomlinson, C. C.
Vinnell, L. F.
Visher, W. A.
Watts, H. M.
Weems, C. M., Jr.
Whitcomb, M. F.
Wilson, H. C.
Wolf, H. S.
Wolpert, M. L.

Central Florida

Bridgland, T. F., Jr.
Dibble, H. L.
Doughty, S. D.
Duval, A. N.
Fritch, S. D.
Gray, A. R.
Houston, D. E.
Houston, E. E., Jr.
Koning, R. E.
Lyden, J. A., Jr.
Masch, H. D.
Mathews, B. E.
Parker, R. L., Jr.
Pelchat, G. M.
Zatlin, F. R.

Florida West Coast

Adkisson, W. M.
Colbert, D. C.
Dingley, E. N., Jr.
Fleischer, K. M.
Goldie, S. P.
Hemmer, F. A., Jr.
Hoffman, R. S.
Landry, W. J.
Langmack, C. E.
Macomber, G. R.
Rosenzvaig, J. E.
Saraydar, R. A.
Simon, L. H.
Streicher, M.
Sullivan, A. W.
Wells, E. L.

Huntsville

Barton, C.
Bradley, B. C.
Finch, M. D.
Gentry, E. B.
Golden, H.
Hallowes, J. P., Jr.
Pittman, W. C.
Schwab, W. G.

Miami

Dillon, J. C., Jr.
Lampkin, G. F.
La Tour, J., Jr.
Owra, W. M.
Payne, V. E.

North Carolina

Cherry, W. H., Jr.
Goetze, A. J.
Lindeman, W. D.
Littleton, W. W.
Padunchewit, I.
Stephens, T. L.
Young, D. B.

Philadelphia

Affel, H. A., Jr.
Aires, R. H.
Alperin, N. N.
Amatneek, K. V.
Anderson, W. G.
Azaren, D.
Bachofer, H. L.
Barger, J. R.
Beck, C.
Benner, A. H.
Benner, R. H., II
Berg, N. E.
Bernstein, F.
Beter, R. H.
Bradshaw, J. L.
Brown, G. M.
Bucsek, G. F.
Buxton, P. T.
Bycer, B. B.
Campanella, M. J.
Canavan, T. P.
Caplan, D. I.
Carpenter, R. A.
Case, C. W.
Cecal, J. A.
Chronister, W. M.
Chudleigh, W. H., Jr.
Cohen, B. H.
Curtin, W. A.
Davidson, R. C.
Davis, R. W., Jr.
Deacon, N. E.
Duetsch, J.
Dickens, B. L.
Dordick, H. S.
Fabbioli, L. F.
Fath, J. P.
Faust, A. C.
Fenton, F. H., Jr.
Fischbeck, K. H.
Foster, J. A.
Friend, A. W.
Fuchs, A. M.
Gluck, S. E.
Goff, K. W.
Gottschalk, J. M.
Greenfield, A.
Gregory, T. R.
Griep, D. J.
Halket, D. R.
Haraldsen, H. P.
Harinett, E. J.
Hayes, H. J.
Hellerman, H.
Herrmann, J. E.
Hoover, E. W.
Huyett, W. I.
Kanal, L. N.
Kashmar, E. J.
Kasowski, S. E.
Kiel, J. H.
Kikuchi, N.
Knipe, W. T.
Kolodner, M.
Kozikowski, J. I.
Krantz, F. H.
Ku, Y. H.
Lathrop, P. A.
La Verghetta, F. E.
Lazinski, R. H.
Levy, A. S.
Lieb, A. B.
Liebermann, J.
Linhardt, R. J.
Lisicky, A. J.
Lockhart, J. C.
Lovett, R. S.
Lowe, T. C.
Macqueene, P. H.
Mandelkehr, M. M.
Mayleben, E. F.
McClure, R. W.
McWilliams, C. R.
McWilliams, G. R.
Milewski, C. A.
Miller, G. F.
Morita, J. T.
Morrow, P. E.
Nines, E.
Osbahr, B. F.
Perecinic, W. S.
Porter, J. W.
Potosky, M.
Price, J. F.
Rao, G. V.
Richter, F.
Riggs, D. L.
Rogers, R. F.
Rogers, R. S.
Roop, R. W.
Rudnick, J. J.
Rudofsky, S.
Rydz, J. S.
Seawell, W. N.
Selinsky, J. J.
Sevian, E. A.
Shahbender, R. A.
Shirman, J.
Shucker, S.
Sink, J. A.
Smith, D. B.
Smith, R. V.
Solomon, S.
Sommer, W. G.
Sorkin, C. S.
Steudel, W. R.
Stout, J. R.
Stubbs, G. S.
Sun, H. H.
Sunstein, D. E.
Sweeney, C. W.
Tonooka, S.
Tweet, B. O., Jr.
Walker, H. R.
Weiner, J. R.
Weisenberger, A. J.
Weiss, E.
Wentz, J. L.
Williams, R. J., III
Willis, J. T.
Wills, W. P.
Wirtz, E. L.
Woerner, L. G., Jr.
Wolfson, H. S.
Wolin, L.
Wolin, S.
Wood, R. F., Jr.
Wooten, L. B.
Yang, T.
Ziegler, I. A.
Zuck, R. A.

Virginia

Andrews, R. E.
Black, J. H.
Branscom, G. A.
Burlingame, C. W.
Cockrell, W. D.
Corpening, A. L.
Dial, E. W.
Eller, J. E., Jr.
Fadeley, J. H.
Gregory, C. A., Jr.
Jones, T. E.
Welch, A. A.

Washington, D. C.

Allen, D. A.
Anders, F. W.
Bernhard, C. J.
Britner, R. O., Jr.
Bush, A. G., Jr.
Bush, G. B.
Carpenter, M. B.
Carruth, D. E.
Chadwell, W. L.
Chen, Y. M.
Chu, Y.
Clarke, A. S.
Collins, J. R.
Coyle, R. J.
Dame, A. M.
Duning, K. E. W.
Enos, R. M.
Finkel, A.
Fleming, J. H.
Fleming, J. J.
Gale, M.
Garofalo, A. D.
George, S. F.
Godsey, W. J.
Herbert, T. O.
Horton, T. R.
James, W. G.
Kelley, J. R.
Kirshner, J. M.
Lee, F. M.
Lee, J. D.
Le Gare, J. M.
Linvill, W. K.
Looney, C. H., Jr.
Mallin, J. A.
Misner, R. D.
Mitchell, G. J.
Morell, C. S.
Morrisey, J. A.
Nickell, W. C.
O'Hara, J. J., Jr.
Ostaff, W. A.
Polak, H.
Ramos, E.
Ray, H. A., Jr.
Reagen, E. J.
Regardh, C. G. B.
Roberts, G. L., Jr.
Rogers, A. L.
Rosen, S. B.
Rozanski, R. R. A.
Sakrison, D. J.
Sanborn, G. D.
Shapiro, G.
Shepard, D. H.
Smith, T. H.
Stoops, C. W.
Talkin, A. I.
Uglow, K. M., Jr.
Varela, A. A.
Viera, F., Jr.
Waterman, P.
Watkins, P. L.
White, C. F.
Wilcomb, E. F.
Wimmer, P. L.
Zastrow, K. D.

Region 4

Akron

Buxton, A. C.
Colletti, N.
Diamantides, N. D.
Flowers, H. L.
Haas, D. L.
Hann, D. D.
Hermann, P. J.
Hurley, W. A.
Lambert, C. O.
Meilander, W. C.
Miller, J. H.
Nelson, R. L.
Penniman, I. B.
Ryburn, P. W.
Stahl, M. D.
Toman, W. J. V.
Yarosh, N. P.
Yochelson, S.

Central Pennsylvania

Bennett, P. E.
Harvey, H. B.
Hoechner, I. L.
Knausenberger, G. E.
Lawther, J. M.
Oblinger, J. T.
Seeley, R. M., Jr.
Wolfe, R. E.

Cincinnati

Berg, D. F.
Blasberg, L. A.
Brown, D. L.
Butterworth, G. S.
Colclaser, R. A.
Dale, W. L.
Doerr, W. H.
Engelmann, R. H.
Forbes, H. R.
Fortier, R. E.
Georger, L. J.
Herrin, C. B.
Hocking, L. J.
Keene, L. C.
Nistico, F.
Sanneman, R. W.

Cleveland

Craig, R. T.
Dambach, R. A.
Frost, E. C.
Gogia, J. K.
Grasson, W., Jr.
Haner, L.
Hart, C. E.
Hickok, R. D.
Hotchkin, E. E.
Kinkaid, J. C.
Kliever, W. H.
Klock, H. F.
Louis, J. R.
Mergler, H. W.
Miller, H. J.
Nadkarni, D. D.
Pfaff, R. W.
Phillips, W. E., Jr.
Saltzer, C.
Shepherd, B. R.
Tame, J. S.

Columbus

Albright, R. E.
Bain, T. D., Jr.
Burgener, R. C.
Chope, H. R.
Cohen, D.
Conlon, R. J.
Cosgriff, R. L.
Cummins, M. M.
Kirschbaum, H. S.
Lahr, R. J.
Lewis, D. E.
McFarland, R. S.
Moll, M.
Newman, A. K.
Spergel, P.
Staats, R. L.
Tamplin, G. R.
Weimer, F. C.

Dayton

Behane, D.
Bornhorst, K. F.
Carr, M. G.
Charbonneaux, W. A.
Cranston, H. J., Jr.
Erdman, B. K.
Fenton, R. E.
Grimm, F. W.
Martino, J. P.
Peterson, L. S.
Simopoulos, N. T.
Thompson, J. P.
Thompson, N. P.
Van Wechel, R. J.
Wichmann, T. F.
Wolaver, L. E.

Detroit

Amber, P. S.
Barcus, R.
Blackwell, W. A.
Bozoian, M.
Brown, L. R.
Brunais, E. G.
Bublitz, A. T.
Burr, H.
Capron, R. W.
Chaney, L. W.
Chow, H.
Chuang, K.
Chute, G. M.
Cummins, D. L.
Gaskell, R. A.
Gilbert, E. G.
Gilbert, E. O.
Harding, W. G.
Ho, Y. C.
Johnson, R. R.
Kazda, L. F.
Klem, R. F.
Koziara, E. H.
Lazarus, D. H.
Lindahl, C. E.
McGlinn, E. J.
McGregor, A. D.
Morgan, B. S., Jr.
Mort, V. A.
Nakagawa, N.
Nixon, J. D.
Olson, R. G.
O'Neal, R. D.
Patton, H. W.
Rauch, L. L.
Rehwoldt, T. V.
Retko, E.
Robinson, D. F.
Roth, P. F.
Scott, D. L. E.
Seleno, A. A.
Sims, R. C.
Smith, B. A.
Smith, D. F.
Smith, W.
Strand, J.
Sutton, W. A.
Taplin, L. B.
Thompson, R. H.
Tokad, Y.
Tubbs, R. J.
Turkish, M. C.
Webber, R. C.
White, O.
Willett, R. M.

Pittsburgh

Aronson, M. H.
Baer, W. E.
Bhavnani, K. H.
Blewitt, D. D.
Boyd, J. J.
Caywood, W. P., Jr.
Chen, K.
Cilyo, F. F., Jr.
Coates, R. S.
Coccia, R. A.
Decker, R. O.
Eggers, C. W.
Ellison, B. P.
Ford, D. J.
Fulmer, L. C.
Golla, E. F.
Hartley, R. R.
Joseph, G. P.
Kinder, H. R.
Little, D. R.
Markel, G. G.
Mathias, R. A.
Moss, A. I.
Mucci, G.
Murray, D. W.
O'Donnell, J. J.
Rau, F. J.
Rogers, L. J.
Schwindt, A. J.
Spriggs, L. A., Jr.
Stankowich, J. A.
Strull, G.
Sze, T. W.
Wissman, A. J.
Wolford, J. E.

Toledo

Butler, J. A.
Ewing, D. J., Jr.
Fuller, L. E.
Leuck, D. D.
Murley, E. M., Jr.
Spademan, C. F.

Williamsport

Erdley, R. F.
Webb, H. E.

Region 5

Cedar Rapids

Anderson, R. L.
Hedgcock, W. T., Jr.
Lowenberg, E. C.
Marsh, C. L., Jr.
McCool, G. W.
McManis, D. L.
Vogt, N. W.

Chicago

Alessio, S. A.
Arndt, L. K.
Auth, L. V., Jr.
Axelrod, L. R.
Barnett, W. T.
Bielenin, A.
Bold, N. T.
Boyd, D. M., Jr.
Brietze, F.

Bullen, C. V.
Burtness, R. W.
Carter, R.
Cermak, C. W.
Chorney, P. L.
Chulsky, I.
Condron, W. F.
Cruz, J. B., Jr.
Davoust, D. J.
Dennis, R. J.
Deterville, R. J.
Dikinis, D. V.
Druz, W. S.
Ebstein, B.
Eilers, C. G.
Fenwick, W. D.
Ferre, G. E.
Foster, G. E.
Fox, A. J.
Frank, D. H.
Fry, J. C.
Fu, K. S.
Furst, U. R.
Gehrke, W. C.
Gerlach, A. B.
Glyptis, N.
Gozner, S. M., Jr.
Greenberg, C. J.
Gregory, E. C.
Heid, K. K. W.
Isolampi, G. E.
Jenness, R. R.
Kortright, F. U.
Kott, W. O.
Kreer, J. B.
Kuchenbecker, R. A.
Lafferty, V. C.
Lesnick, R.
Levine, M. L.
Levy, G. F.
Lewis, H. A.
Li, C. C.
Ma, H. J.
Martin, J. W., Jr.
Masteller, B. L.
Mercure, R.
Merrifield, L. A.
Meyer, A.
Mittelmann, E.
Motch, J. F.
Newman, J. B., Jr.
Noges, E.
Olson, J. C.
Paterson, W. L.
Rebman, W. H.
Rockwood, C. C.
Ruther, H. E.
Sayles, H. L.
Severns, R. A.
Shewan, W.
Sigborn, T. W.
Slana, M. F.
Sommeria, M. R.
Thielen, L. R.
Thomas, R. G.
Van Bosse, J. G.
Vandling, G. C.
Van Ness, J. E.
Van Valkenburg, M. E.
Veazie, W.
Venne, R. E.
Veras, J. A.
Verbanec, W. R.
Vitous, J. P.
Warshawsky, J.
Waters, J. I.
Wavering, A. J.
Weissman, R. M.
White, E. S.
Woolums, L. L.
Ye, J. W. F.

Fort Wayne

Brady, F. H.
Cronkwright, R. E.
Emery, R. C.
Fife, D. W.
Johnson, D. L.
Kaplan, R.
Klingler, E. H.
Mason, C. F.
Nelson, G. L.
Richeson, W. E.
Solomon, R. M.
Williams, W. J., Jr.

Indianapolis

Albon, R., Jr.
Braunagel, M. V.
Cannon, C. D.
Carpenter, M. M., Jr.
Cross, K. R.
Ennis, F. L.
Evans, R. A.
Gibson, J. E.
Groves, C. R.
Halsey, M. R.
Israel, J. D.
Johnson, T. L.
Jones, M. E.
Luisi, J. A.
Marshall, H. B.
Ogborn, L. L.
Skinner, G. M.
Spencer, J. L.
Tou, J.
Whipple, R. L.
White, S. A.

Louisville

Ebaugh, D. P.
Kain, R. Y.
Kwo, T. T.

Milwaukee

Arakelian, G. P.
Bennett, T. H.
Camps, T. F.
Carlson, A. W.
Diggelmann, H.
Frankos, D. T.
Gessner, U.
Graham, J. D.
Jensen, K. S.
Kerske, J. F.
Laurents, V. T.
Leary, R. W.
Limpel, E. J.
Lind, E. R.
Lindemann, A. W.
Lofy, F. J.
Louzader, J. C.
Mayer, H. J.
Mezger, J. P.
Morin, D. C., Jr.
Neider, F. V.
Norum, V. D.
Pierce, R. L.
Pierick, K. R.
Rekoff, M. G., Jr.
Smith, C. C.
Stephan, R. R.
Weissenborn, H. E.
West, B. M.

Omaha-Lincoln

Bashara, N. M.
Terry, R. W.

South Bend-Mishawaka

Burke, T. H.
Bymberg, R. J., Jr.
Chen, G. K. C.
Cooney, J. J.
Hansen, A. G., Jr.
Hoffman, C. H.
Minnick, V. P.

Twin Cities

Adams, G. E.
Alderson, R. C.
Alfsen, G. F.
Alterman, F. J.
Balzart, E. J., Jr.
Bartlett, V. W.
Benassi, D. A.
Bergan, K. N.
Burns, S. S.
Carlson, R. A.
Evans, J. M.
Gise, F. G., Jr.
Gustafson, H. A.
Hardenbergh, G. A.
Harvey, P. O.
Hird, F. S.
Hulstrand, B. E.
Inman, T. F.
Kershaw, J. A.
Kiene, R. C.
Knoblauch, A., Jr.
Kukuk, H. S.
Lahue, P. M.
Lanzkron, R. W.
Lease, L. R.
Lode, T.
Ludwig, J. T.
Luik, I.
Markusen, D. L.
Maze, R. O.
McLane, R. C.
Moe, W. J.
Murray, R. L.
Nellis, W. M.
Noble, D. S.
Nordstrom, J. E.
Ormsby, R. D.
Pierce, A. J.
Ramamoorthy, C. V.
Reed, M. W.
Rowland, C. A., Jr.
Schellenberg, A. G.
Schiebe, F. R.
Schuck, O. H.
Senstad, P. D.
Smith, D. A.
Stone, N. T.
Storm, J. F.
Swanholm, W. J.
Swanlund, G. D.
Vogel, J. P.
Windsor, R. N.
Woll, L. J.

Region 6

Alamogordo-Holloman

Carlson, H. A.
Chandler, D. P.
Koellner, K. H.
Liston, D. H.
Pykosz, T. F.

Dallas

Anderson, R. P.
Askew, W. J., Jr.
Bailey, R. W.
Barnett, M. L.
Braun, C.
Buehrle, C. D.
Dubose, G. P.
Gentry, J. A.
Humke, F. O., Jr.
McDonald, M.
Miller, N. D.
Osnaschek, F. J.
Pittman, P. D., Jr.
Scammel, B. C.
Sparkman, W. A., Jr.
Stanton, A. N.
Tatum, F. W.
Teasdale, A. R., Jr.
Wadel, L. B.
Wilhelm, E. S.
Ziemer, D. R.

Denver

Bergman, R. R.
Capehart, M. E.
Cook, E. E.
Daniels, W. H.
Hart, W. G., Jr.
Howell, R. G.
Messler, F. J.
Mielziner, W.
Morroni, D. J.
Ostwald, L. T.
Spafford, L. J.
Stacey, D. S.

El Paso

Bullock, R. H.
Martin, T. A.
Moore, T. E.
Moss, E. C.
Rojas, A. M. H.

Fort Worth

Delaney, J. S.
Evans, W. L.
Forester, J. R.
Hines, R. L.
Jiles, C. W.
Jones, L. R.
Perry, E. R.
Seale, I. A.
Simmons, D. J.
Slagle, G. M., Jr.
Watkins, O. E.

Houston

Bucy, J. F., Jr.
Eads, R., Jr.
Easterling, M. F.
Frobese, C. W.
Groendycke, A. R.
Hargett, J. T.
Keating, L. M.
Keeling, W. E.
Kuykendall, W. E.
Lockwood, R. M.
Navarro, S. O.
Sherrill, B.
Tasini, B.
Terao, M.

Kansas City

Annis, J. W.
Brown, G. M.
Clum, L. E.
Halijak, C. A.
Lobb, C. W.
Rauch, E. L.
Reiser, R. R.
Roehm, R. A.
Simonds, R. L.
Stout, H. L.
Wilcox, J. V.

Little Rock

Anthony, C. A.
Bodie, M. W.
Butler, B. L.
Cannon, W. W.

Lubbock

Perkins, C. S.

New Orleans

Cronvich, J. A.
Dietz, W. R.
Drake, R. L.
Hodge, B.
Maus, L. C.
Smith, L. G.
Weiser, C. C., Jr.
Welsh, M. J., Jr.

Oklahoma City

Glennon, R. E.
Grubbs, C. E.
Ledbetter, R. P.
Vlay, G. J.

St. Louis

Abkemeier, L. R.
Barr, R. K.
Cassidy, R. E.
Dawson, R. T.
Ebert, W. J., Jr.
Fiedler, G. J.
Hamilton, S.
Herchenreder, H.
Hudgens, L. L.
Lago, G. V.
Malsbary, J. S.
Mayer, M. F., Jr.
McAninch, C. H.
Min, H. S.
Mohrman, R. F.
Mutchek, J. H.
Norman, C. F.
Palmer, J. A.
Reed, D. L.
Sayer, J. D.
Scherz, C. J.
Sheehan, J. S.
Sudfeld, C. C.
Twombly, J. W., Jr.
Weaver, M. D.
Zaborszky, J.

San Antonio

Bostick, F. X.
Fisher, H. V., Jr.
Hirsch, C. O.
Hoffman, A. A. J.
Phillips, J. P.
Reinhard, E. A.
Saadeh, D.

Shreveport

Averre, C. W.
Gordon, E.
Long, F. V.

Tulsa

Brashear, R. T.
Day, C. E.
Freeman, L. R.
Fox, D. N.
Greening, J. P.
Labarthe, L. C.
Peterson, G.
Piety, R. G.
Rowley, R. G., Jr.
Silverman, D.
Sykora, G. E.
Wolkov, D.

Region 7

Albuquerque-Los Alamos

Baker, D. A.
Carlson, W. F.
Foster, F. N.
Helgeson, B. P.
Hu, C. T.
Koschmann, A. H.
Koskela, A. C.
Pace, T. L.
Powell, R. L.
Ray, H. K.
Tosti, D. T.

China Lake

Creusere, M. C.
Dolce, S. L.
Dorsey, S. E.
Glatt, B.
Kim, P. K. S.
Poulson, W. A.
Starr, E. A.

Los Angeles

Adomian, G.
Akers, R. C.
Akin, P. A.
Albrecht, A.
Alfrey, P. R.
Allen, D. H.
Alper, S. M.
Ambrose, J. R.
Amstutz, M. F.
Anderson, F.
Anderson, M. J.
Anderson, W. E.
Andrews, L. V. A.
Anzel, B. M.
Arnold, J. R.
Aroyan, G. F.
Arsenault, W. R.
Aseltine, J. A.
Auletti, R. L.
Avrech, N.
Baker, D. L.
Baker, J. N.
Barlett, F. R.
Barnett, L.
Barston, E. M.
Bartholomew, H. R.
Beard, G. W.
Beckwith, H. W.
Bekey, G. A.
Belding, R. A.
Bement, W. A.
Bemis, R. C. B.
Berg, R. L., Jr.
Berman, M. R.
Bertorello, R. J.
Bible, R. E.
Bills, G. W.
Bonney, R. B.
Borgeson, P. W.
Borsch, K. S.
Bower, J. L.
Brandt, R.
Braun, E. L.
Braverman, D. J.
Brewer, H. E.
Broadwell, W. B.
Bronstein, L. M.
Brown, D. E.
Bucher, F. X.
Buland, R. N.
Bunkowski, N. H.
Burk, W. A.
Burnsweig, J., Jr.
Byck, D. M.
Cahn, J. M.
Callot, S.
Campbell, G., Jr.
Capps, J. W.
Carlson, C. O.
Chandaket, P.
Chang, B.
Chapman, C. W.
Chernoff, D. P.
Christensen, A. V.
Cochran, E. D.
Concus, P.
Conway, J. P.
Cooke, C. W., Jr.
Corley, C. F.
Cornell, J. R.
Corvi, J. A.
Cosgrave, S. J.
Craven, W. A., Jr.
Crowe, J. W.
Curry, W. S., Jr.
Cutting, E.
Dains, H. O.
Davis, H. G., Jr.
Day, R. N.
Deaux, F. J.
Deen, J. R.
Deming, A. F.
Determan, J. D.
Deuser, D. A.
Di Bias, R. L.
Dickinson, H. B.
Diem, C. W.
Diemer, F. P.
Dietz, D. P.
Di Giuseppe, A.
Dinning, J. R.
Doty, R. L.
Drucker, A. N.
Dzilvelis, A. A.
Edelsohn, C. R.
Eggeman, D. J.
Eichwald, W. F.
Eikelman, J. A., Jr.
Ellis, D. O.
Engel, H. L.
Eno, R. F.
Epstein, S.
Estrin, G.
Evfimenko, A.
Finley, W. A.
Fischer, P. F.
Fish, K. L.
Fish, W. Y.
Forbath, F. P.
Fox, C. W.
Foxman, E.
Francis, T. F.
Furumoto, N.
Gabler, R. T.
Garber, L. F.
Garcia, E. M.
Gardner, F. H.
Gardner, F. M.
Gauronskas, P. P.
Gaylord, R. S.
Gee, L. C.
Gerken, G. H.
Gill, W. J.
Glowalla, J.
Goldstein, A.
Gore, M. R.
Grabbe, E. M.
Graham, J. D.
Green, R. D.
Greensberg, E. L.
Gronstrand, T. L.
Gross, W.
Gullatt, S. P., Jr.
Gunning, W. F.
Gustin, J. T.
Hadden, F. A.
Hailey, R. D.
Hall, C. R.
Halloran, R. G.
Hanna, R. E.
Harmon, W. G.
Hassel, R. R.
Hawkins, J. K.
Hayes, J. E.
Helland, J. C.
Heyliger, G. E.
Hicks, A. R.
Hinrichs, K.
Hitchcock, R. W.
Holmen, R. E.
Hom, F. M.

Hoskinson, E. A.
Hoving, H. R.
Howard, C. W.
Hoy, E. C.
Hruby, R. J.
Hunt, E. B.
Hurley, R. F.
Hutcheon, R. S.
Izuel, A. G.
Jack, R. W.
Jackson, C. F.
Jackson, K. R.
Jacobs, J. E.
Jacobson, O. M.
Janeway, R. K.
Jenkins, L. E.
Jeschke, A. W.
Joerger, J. C.
Johnson, M. G.
Johnson, W. A.
Jones, J. M.
Kahn, B. S.
Kaufman, F. H.
Kaufman, S.
Kawahata, B. I.
Kazarian, D. G.
Kennedy, C. J.
Kennedy, F. D.
Kennel, J. M.
Keppel, R. A.
Kern, W. W.
Kerster, G.
King, C. G., Jr.
King, J. E.
Kirk, C. N.
Kirsch, H. A.
Kishi, F. H.
Kitabayashi, T.
Klimowski, F., Jr.
Knox, R. V.
Kobayashi, R.
Krill, C. K.
Kroy, W. H., Jr.
Kuerschner, H.
Kukel, J.
Kuntz, G. L.
Lawrence, A. F., III
Lebell, D.
Lee, H. J.
Leondes, C. T.
Leone, W. C.
Lessley, T. D.
Levine, L.
Levine, S. E.
Levinson, R. M.
Levy, E. C.
Lewis, D. E.
Lichterman, G.
Lillibridge, E. H.
Lindholm, C. R.
Lloyd, A. T.
Lords, F. V.
Louie, W.
Lyon, D. B.
Lyons, L. H.
Makush, W. S.
Mancini, A. R.
Manly, R.
Manser, D. E., Jr.
Margolis, M.
Maticich, J. R.
Mayberry, L. A.
Mayner, G. S.
McCarthy, J. W.
McCormick, G. F.
McCann, L.
McGhee, R. B.
McLarin, M.
McLeod, M. G.
McRuer, D. T.
McVey, I. M.
Mehner, E. W.
Metzner, H. E.
Mileson, D. F.
Miller, D. S.
Mitsutmoi, T.
Modlinski, W. M.
Moise, N. L.
Monroe, F. R.
Moore, P. G.
Morris, G. R.
Morrison, A. I.
Morton, W. B., Jr.
Mosher, W. W., Jr.
Moss, J. F., Jr.
Mundt, E.
Myers, W. A.
Nedland, E. H.
Nelson, C. S., Jr.
Noda, M.
Noland, A. R.
Nuban, E.
Nussbaum, O. N.
Nuttall, H. V.
O'Brien, W. C.
Odlum, F. H., Jr.
O'Hara, C. L.
Ohrenstein, S. B.
Olsen, J. D.
Olsen, L. V.
Parker, A. T.
Parzl, R. C.
Patrusky, N.
Perez, A. A.
Perkins, L. M.
Pernick, L. J.
Peterson, E. J.
Peterson, R. W.
Phister, M., Jr.
Poirier, J. P. A.
Post, G.
Primozich, F. G.
Pullen, E. W.
Putnam, B. G.
Quackenbush, R. E.
Quinn, T. B.
Radant, M. E.
Raffensperger, M. J.
Ramer, F. H., Jr.
Ramstedt, C. F.
Rea, D. E.
Rea, W. P., Jr.
Redden, E. T.
Redmond, J. G.
Rehler, K. M.
Rescoe, J. M.
Rickords, T. J.
Rieman, F. C.
Rifkind, J.
Robinson, J. A.
Rogers, J. G.
Rogers, J. M.
Rogers, T. A.
Romano, A. J.
Rosenberg, R. E.
Rosenstein, A. B.
Ross, I.
Ruiz, M. L.
Russell, W. J.
Russell, W. T.
Saks, S. A.
Salmi, T. W.
Samuels, A. H.
Sanderson, K. W.
Sarture, C. W.
Savant, C. J., Jr.
Savo, T. A.
Sawyer, H. F.
Schalk, N.
Schmidt, L. O.
Schnauss, E. R.
Schneider, R. L.
Schneider, S.
Schnoor, J. E.
Schroeder, W.
Schull, G. R.
Schulte, R. W.
Schultz, P. R.
Schultz, R. T.
Schulz, K. S.
Scope, S.
Scott, R. G.
Scott, W. F.
Scott, W. H., Jr.
Sear, A. W.
Sensiper, S.
Shelley, R. G.
Shenk, J. W.
Shimada, G.
Shuler, M. H.
Shultise, Q. M.
Shutt, S. G.
Sibilio, R. A.
Siegel, J. C.
Silva, L. M.
Simmons, J. C.
Simpson, B. L.
Sink, R. L.
Sizemore, L. E.
Slocomb, G. M.
Smith, B. N.
Smith, C. C. L.
Smith, J. D.
Smith, L. C.
Smith, W. B.
Snapp, K. M.
Snyder, W. A.
Sofen, I. A.
Sohler, J. F.
Sokol, J. L.
Squires, W. K.
Staudhammer, J.
Stear, E. B.
Steinkolk, R. B.
Stephenson, P. L.
Stout, T. M.
Stow, R. A.
Sturm, T. F.
Sundberg, R. A.
Swanson, G. W.
Swarthe, E.
Takahashi, K.
Tallman, G. H.
Tanaka, R. I.
Tanner, W. O., Jr.
Taylor, J. C.
Telle, G. R.
Tesler, A.
Thomas, R. L.
Thomson, H. C.
Thorenson, R.
Tomlinson, Z. G.
Tracey, B. P.
Tucker, M. C.
Turn, R.
Udry, J. J.
Uyehara, M.
Vail, W. A.
Valery, N. A.
Valiquette, D. J.
Van Curen, V.
Vega, C. J.
Vittum, W. M.
Vodovoz, E.
Vulliet, P. O.
Wachowski, H. M.
Wakamiya, Y.
Walker, E. S.
Walp, R. M.
Walsh, J. F.
Walters, L. G.
Wanlass, S. D.
Watkins, E. L.
Wedel, J. J., Jr.
Wells, G. H.
Wennerberg, G.
Wenters, R. L.
Wertheim, L. A.
Whelan, D. E.
White, L. M.
Whitford, R. K.
Williams, H. M.
Wilson, G. P.
Wolman, L. L.
Wong, D. S.
Wong, E. C.
Woo, S. H.
Wunderlich, F. J.
Wyatt, W. C.
Yamada, D. A.
Yamamoto, J. R.
Yamamoto, T. G.
Yoshii, H.
Young, W. L.
Zabusky, N. J.
Zacharias, R.
Ziegler, R. M.
Zimmerman, R. L.
Zoller, C. J.

Phoenix

Ballantine, J. H.
Clark, P. S.
Donovan, J.
Gaines, W. M.
Herrera, E. B.
Hodson, R. B.
Ittenbach, L. J., Jr.
Levine, D.
Mayes, T. L., Jr.
Montgomery, E. B.
Richman, M. A.
Ross, J. M.
Sabol, R. W.
Scrafford, R. L.
Simpson, E. J.

Portland

Deer, J. W.
Doel, D.
Hinmers, K. J.
Jenkins, R. W.
Mendoza, D. C.
Stone, L. N.

Sacramento

Akrivos, G. D.
Cordray, R. E.

Salt Lake City

Hammond, S. B.
Heiser, R. K.
Jackson, H. L.
Johnson, W. L.
Linebarger, R. N.
Lubeck, R. V.
McCollom, K. A.
Warner, B. D.

San Diego

Bayley, L. B., Jr.
Biering, A. H.
Boelens, J.
Campbell, R.
Cox, T. M. E.
Dodd, G. M.
Evans, W. O.
Ferner, R. O.
Flarity, W. H.
Fogel, L. J.
Hallman, A. B.
Hardy, W. G.
Herman, J. J.
Hodges, P.
Holcomb, D. E., Jr.
Kalbfell, D. C.
La Gue, T. L.
Loeb, M.
Mealey, G. J.
Michael, F. J.
Murray, J. H.
Norris, B. J.
Prager, R. H.
Scarborough, C. S.
Schneebeck, D. A.
Shechet, M. L.
Stone, K. A.
Tamura, Y.
Waddell, B. L.
Wade, E.
Weisbrod, S.

San Francisco

Ackerlind, E.
Allison, J. E.
Arnold, D. T.
Bahrs, G. S.
Barnard, G. A., III
Bean, D. A.
Beatie, R. N.
Becher, W. D.
Bently, D. E.
Berryhill, J. L.
Bharucha, B. H.
Binnall, E. P.
Bliss, J. C.
Boennighausen, R. A.
Brennan, R. D.
Bridgman, A. D., Jr.
Brooks, H. B.
Brooks, R. E.
Brunetti, C.
Bullock, J. B.
Buntenbach, R. W.
Bunzl, T. F.
Buss, R. R.
Carnahan, C. W.
Carter, J. M.
Chesebro, E. L.
Cortes, A.
Cox, J. F.
Davy, L. H.
De Bra, D. B.
De Tolly, G. E. B.
Durfey, G. K.
Ehrenburg, F. K.
Elster, S.
Farman-Farmaian, G.
Finnigan, R. E.
Firschein, O.
Fisher, R. C.
Flugge-Lotz, I.
Franklin, G. F.
Gagnon, R. J.
Gardiner, K. W.
Glover, R. D.
Goslow, P.
Griffith, P. G.
Guilford, E. C.
Gyllstrom, N. D.
Halina, J. W.
Halvorsen, J. L.
Hammond, G. P., Jr.
Henry, E. W.
Hexem, J.
Hopkin, A. M.
Horwitz, L. B.
Huelsman, L. P.
Humphries, J.
Hunt, J. M.
Hutton, L. G.
Iwama, M.
Jameson, R. J.
Jesse, E.
Johnson, R. W.
Jury, E. I.
Katt, D. R.
Kazanjian, H. A.
Kerwin, W. J.
Kirkland, J. A.
Klotter, K.
Kochenderfer, W. E., Jr.
Koski, T. H.
Kritzer, E. A.
Lally, J. P.
Leavitt, C. W.
Lee, T. W.
Lendaris, G. G.
Lindberg, H. E.
Linden, D. A.
Linders, T. E.
Littler, D.
Lohr, D.
Lusk, T. D.
Mace, J. C.
MacGinitie, G.
Mancuso, W.
Matson, D. L., Jr.
Mattingly, M. L.
McCullough, W. R.
Meisling, T. H.
Monnier, R. E.
Moorehead, D. L.
Mullin, F. J.
Muntz, W. E.
Nickel, L.
Nunamaker, T. A.
Oliver, B. M.
Park, G. L.
Peters, R. L.
Pitsenbarger, G. A.
Plotkin, S. C.
Pope, J. C.
Regenos, K. M.
Roberts, T. E., Jr.
Rodgers, P. W.
Rolph, D. B.
Ross, A.
Samario, E. J.
Santana, G. R.
Saunders, R. M.
Scott, D. G.
Shephard, R. W.
Short, R. A.
Sierra, H. M.
Singer, J. R.
Smith, D. L.
Smith, O. J. M.
Spilker, J. J., Jr.
Stephenson, J. M.
Stoll, P. J.
Stoltz, J. R.
Storke, F. P., Jr.
Sullivan, J. M., Jr.
Tavernia, G. B.
Thal-Larsen, H.
Thomas, R. E.
Thomasian, A. J.
Treseder, R. C.
Tuttle, D. F., Jr.
Vea, T. H.
Wang, P. K. C.
Watson, J. K.
Woo, J.
Wye, R. E.

Seattle

Bergseth, F. R.
Biggs, J. D., Jr.
Birch, J. S.
Bishop, D. J.
Boys, J. A.
Brock, R. L.
Clark, R. N.
Duggan, J. E.
Farris, W. E.
Galloway, W. C.
Gibson, G. A.
Graybeal, J.
Head, G. M.
Kieburtz, R. B.
Kedray, W.
Lindsley, J. C.
Miller, J. J., Jr.
Noland, L. J.
Seilstad, N. D.
Sharp, B. M.
Skahill, B. J., Jr.
Stathacopoulos, A. D.
Tate, J. A.
Tenning, C. B.
Tufts, W. F.
Vermilion, E. E.
Wall, R. E., Jr.
Whipple, M. M.
Wolff, E. R.

Tucson

Bard, W. E.
Britton, J. W., Jr.
Herndon, T. R.
Leifheit, S. E.
Lindenberg, E. C.
Martin, L. C.
Pilling, L. H.
Wilde, N.
Wright, K. F.

Region 8

Bay of Quinte

Flemons, R. S.
MacKelvie, J. S.
Young, J. A. I.

Hamilton

Grein, A. E.
Kassner, J.
Kennedy, H. G.
Kozak, W. S.
Valian, T. L.
Watts, T. O.

London

Fletcher, H. R.
Stroud, E. L.

Montreal

Bar-Urian, M.
Baumans, H. W.
Birman, G.
Caron, J. Y.
Cote, G. R.
Cox, J. R. G.
Cummins, J. A.
Delisle, J. O.
Dingwall, R. A.
Dinovitzer, N. A.
Germain, L. V.
Gravel, J. J. O.
Harper, M. J.
Heckman, G. R.
Kingan, A. J.
Komaroff, J. G.
Lee, P. J. L.
Maclennan, N. D.
Mahoney, J. A.
Martin, D. M.
Prichodjko, A.
Rasmussen, F. H.
Reeves, R.
Richard, G. B.
Rinfret, C. J.
St. Onge, J. L.
Wood, H. H.
Zaitlin, B. J.
Zames, G.

Northern Alberta

Carle, D. W.
Carlisle, D. M.
Doherty, J. A.
Goddard, G. E.
Little, S. B.

Ottawa
Barrigar, R. H.
Beneteau, P. J.
Chrzanowski, J. T.
Clemence, C. R.
Cowper, G.

Regina
McGuffin, J. W.

Southern Alberta
Johnston, C. W.
MacDonald, A. G.

Toronto
Baldwin, J. H.
Byers, H. G.
Cade, P. G.
Campbell, J. H.
Carew, S. J. H.
Carley, R. R.
Coats, W. D.
Davy, A. S.
Hackbusch, R. A.
Herzog, S. L.
Kipiniak, W.
Lang, G. R.
Leslie, J. D.
McClean, R. K.
McCloskey, K. P.
Morden, R.
Newhall, E. E.
Otsuki, J. S.
Paterson, D. G.
Penrose, R. M.
Shortt, A. J.
Turner, N. P.
Wall, E.

Vancouver
Bohn, E. V.
Hewit, H. O.
Jezioranski, J.
Kersey, L. R.
McKimm, T. F.
Moore, A. D.
Pirart, M. A.
Thom, D. C.
Ward, R. A.

Winnipeg
Milton, J. S.
Shapera, G. H.

Foreign Sections

Argentina
Pinasco, S. F.

Brazil
De Mattos, H. C.

Egypt
Kamal, A. A.

Israel
Bonenn, Z.
Kreindler, E.
Mass, J.
Shamir, J.
Weislitzer, J.

Japan
Aoi, S.
Ezoe, H.
Harada, N.
Harashima, O.
Hata, S.
Honda, T.
Ibuka, M.
Imai, H.
Ishikawa, T.
Iwakata, H.
Kitsuregawa, T.
Kohno, S.
Koichibara, T.
Kojima, Y.
Konomi, M.
Kumagai, M.
Matsuyuki, T.
Mikuma, F.
Minozuma, F.
Miyakoshi, K.
Morita, K.
Morita, M.
Nakagami, M.
Nishino, O.
Niwa, Y.
Okada, M.
Okamura, S.
Owaki, K.
Ozaki, H.
Saito, Y.
Shintani, T.
Tanabe, Y.
Taniguchi, F.
Togino, K.
Yamazaki, T.
Yano, A.

Foreign Countries

Australia
Brodribb, M. I.
Davies, R. J. C.
Honnor, W. W.
Khor, T. H. M.
Willoughby, E. O.

Belgium
Helbig, W. L.
Hoffmann, J. A. J. L.
Murphy, B.

Brazil
Barros-Barreto, L. A. G. C.
Chow, Y.
Kida, S.
Waeny, J. C. C.

Chile
Ribbeck, C. S.

Cuba
Miller, K. E.

Denmark
Marhauer, H. H.
Overlie, P. T.
Sonne, E. G.

England
Booth, P. C.
Cattanes, E.
Clark, J. E.
Cullen, A. L.
Dastidar, P. R.
Dawes, E. J.
Dietiker, W.
Funke, E. R. R.
Gagne, R. E.
Harris, K. E.
Higham, E. H.
Jackson, W.
Laverick, E.
Robertson, A. A.
Rose, S. P.
Szentirmai, G.
Warr, H. J. J.

Finland
Luoto, U. A.

France
Baron, J.
Berline, S. D.
Ferrier, P. A.
Floquet, J.
Fuks, A.
Ghertman, J.
Girerd, J. L. M.
James, R. M.
Labin, E.
Lermoyez, M. J.
Loeb, J. M.
Mandel, P.
Oudard, A.
Simon, J. C.

Germany
Busch, C. W.
Effertz, F. H.
Peters, J. F.
Rohde, L.
Walther, A.

Greece
Icihhakis, M. A.
Papadopoulos, N. D.

India
Ganapaty, S.
Kurma, V. R.
Mishra, S.
Mukerji, M. M.

Italy
Alberti, A.
Bacchialoni, F. L.
Balzano, G.
D'Auria, G.
De Dominicis, C. M.
Derossi, A. D.
De Vito, G. R.
Egidi, C.
Ercoli, P.
Fagnoni, E.
Floriani, V.
Missio, D. V.
Palandri, G. L.
Pinolini, F.
Quaglia, G.
Tchou, M.
Tiberio, U.
Valentini, V.
Verdoni, L. G.
Vergani, A.

Lebanon
Hoffman, J. D.

Mexico
Auerbach, L. F.
Higuera-Mota, H. R.
Lepe-Ramos, F.
Marenco, E. G.
Rodriguez, E.

Netherlands
Alma, G. H. P.
Janssen, J. M. L.
Tellegen, B. D. H.

Norway
Engvik, S. B.

Romania
Tanasescu, T. A.

Scotland
Mascall, A. L.

South Africa
Zawels, J.

Sweden
Andersson, K. N.
Ekelof, S.
Elfving, A. L.
Fagerlind, S. G.
Gyllenkrok, T. G.
Hartman, D. A. B.
Josephson, B. A. S.
Laurent. T.
Lindstrom, G.
Lofgren, E. O.
Perers, O. F.
Persson, N. I. E.
Roll, A.
Romell, G. D. R.
Sivers, C. H. V.
Svala, C. G.
Wikland, T. E.

Switzerland
Braun, A. F.
Nordby, K. S.
Roch, A. A.
Shah, R. R.
Strohschneider, W.
Thalmann, V.
Weber, G. C.

USSR
Kostanjanz, B. A.
Sobolev, M. A.

Venezuela
Arreaza, R. G.
Bartelme, R. R.

Military Overseas
Britto, J. D.
Egan, T. R.
Liston, J.
Rearick, H. F.
Schumacher, G. B.

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Authors wishing to present papers at the 1958 IRE WESCON Convention to be held in Los Angeles, Calif., August 19–22, should send 100-word abstracts and either the complete text or a detailed summary to the Technical Program Committee Chairman:

Dr. Robert C. Hansen
Microwave Laboratory
Hughes Aircraft Co.
Culver City, Calif.

There will be again an IRE WESCON CONVENTION RECORD. Authors will be notified of acceptance or rejection by June 1, 1958.